3

Fractions and Mixed Numbers

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from Campus to Careers

School Guidance Counselor

School guidance counselors plan academic programs and help students choose the best courses to take to achieve their educational goals. Counselors often meet with students to discuss the life skills needed for personal and social growth. To prepare for this career, guidance counselors take classes in an area of mathematics called *statistics,* where they learn how to collect, analyze, explain, and present data.

In **Problem 109** of **Study Set 3.4,** you will see how a counselor must be able to add fractions to better understand a graph that shows students' study habits.

JOB TITLE:
School Guidance Counselor

EDUCATION: A master's degree is usually required to be licensed as a counselor. However, some schools accept a bachelor's degree with the appropriate counseling courses.

JOB OUTLOOK: Excellent.

ANNUAL EARNINGS: The average (median) salary in 2006 was $53,750.

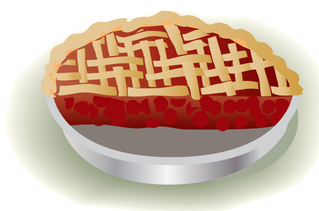
FOR MORE INFORMATION:
www.bls.gov/oco/ocos067.htm

Objectives

1. Identify the numerator and denominator of a fraction.
2. Simplify special fraction forms.
3. Define equivalent fractions.
4. Build equivalent fractions.
5. Simplify fractions.

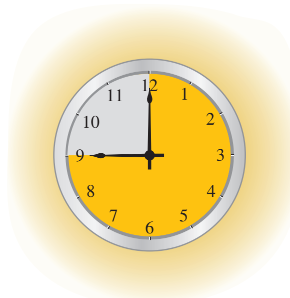
SECTION 3.1
An Introduction to Fractions

Whole numbers are used to count objects, such as CDs, stamps, eggs, and magazines. When we need to describe a part of a whole, such as one-half of a pie, three-quarters of an hour, or a one-third-pound burger, we can use *fractions*.



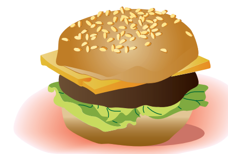
One-half of a cherry pie

$\frac{1}{2}$



Three-quarters of an hour

$\frac{3}{4}$



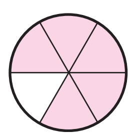
One-third pound burger

$\frac{1}{3}$

1 Identify the numerator and denominator of a fraction.

A **fraction** describes the number of equal parts of a whole. For example, consider the figure below with 5 of the 6 equal parts colored red. We say that $\frac{5}{6}$ (five-sixths) of the figure is shaded.

In a fraction, the number above the **fraction bar** is called the **numerator,** and the number below is called the **denominator.**



Fraction bar ⟶ $\frac{5}{6}$ ⟵ numerator / ⟵ denominator

The Language of Mathematics The word *fraction* comes from the Latin word *fractio* meaning "breaking in pieces."

Self Check 1

Identify the numerator and denominator of each fraction:

a. $\frac{7}{9}$

b. $\frac{21}{20}$

Now Try Problem 21

EXAMPLE 1 Identify the numerator and denominator of each fraction:

a. $\frac{11}{12}$ **b.** $\frac{8}{3}$

Strategy We will find the number above the fraction bar and the number below it.

WHY The number above the fraction bar is the numerator, and the number below is the denominator.

Solution

a. $\frac{11}{12}$ — 11 ⟵ numerator, 12 ⟵ denominator

b. $\frac{8}{3}$ — 8 ⟵ numerator, 3 ⟵ denominator

If the numerator of a fraction is less than its denominator, the fraction is called a **proper fraction.** A proper fraction is less than 1. If the numerator of a fraction is greater than or equal to its denominator, the fraction is called an **improper fraction.** An improper fraction is greater than or equal to 1.

Proper fractions

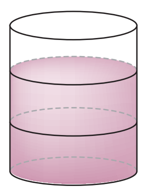
$\frac{1}{4}$, $\frac{2}{3}$, and $\frac{98}{99}$

Improper fractions

$\frac{7}{2}$, $\frac{98}{97}$, $\frac{16}{16}$, and $\frac{5}{1}$

The Language of Mathematics The phrase *improper fraction* is somewhat misleading. In algebra and other mathematics courses, we often use such fractions "properly" to solve many types of problems.

EXAMPLE 2 Write fractions that represent the shaded and unshaded portions of the figure below.



Strategy We will determine the number of equal parts into which the figure is divided. Then we will determine how many of those parts are shaded.

WHY The denominator of a fraction shows the number of equal parts in the whole. The numerator shows how many of those parts are being considered.

Solution

Since the figure is divided into 3 equal parts, the denominator of the fraction is 3. Since 2 of those parts are shaded, the numerator is 2, and we say that

$\frac{2}{3}$ of the figure is shaded. Write: $\frac{\text{number of parts shaded}}{\text{number of equal parts}}$

Since 1 of the 3 equal parts of the figure is not shaded, the numerator is 1, and we say that

$\frac{1}{3}$ of the figure is not shaded. Write: $\frac{\text{number of parts not shaded}}{\text{number of equal parts}}$

Self Check 2

Write fractions that represent the portion of the month that has passed and the portion that remains.

DECEMBER

~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~7~~
~~8~~	~~9~~	~~10~~	~~11~~	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Now Try **Problems 25 and 101**

There are times when a negative fraction is needed to describe a quantity. For example, if an earthquake causes a road to sink seven-eighths of an inch, the amount of downward movement can be represented by $-\frac{7}{8}$. Negative fractions can be written in three ways. The negative sign can appear in the numerator, in the denominator, or in front of the fraction.

$$\frac{-7}{8} = \frac{7}{-8} = -\frac{7}{8} \qquad \frac{-15}{4} = \frac{15}{-4} = -\frac{15}{4}$$

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Notice that the examples above agree with the rule from Chapter 2 for dividing integers with different (unlike) signs: *the quotient of a negative integer and a positive integer is negative.*

2 Simplify special fraction forms.

Recall from Section 1.5 that a fraction bar indicates division. This fact helps us simplify four special fraction forms.

- ***Fractions that have the same numerator and denominator:*** In this case, we have a number divided by itself. The result is 1 (provided the numerator and denominator are not 0). We call each of the following fractions a **form of 1.**

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \frac{7}{7} = \frac{8}{8} = \frac{9}{9} = \dots$$

- ***Fractions that have a denominator of 1:*** In this case, we have a number divided by 1. The result is simply the numerator.

$$\frac{5}{1} = 5 \qquad \frac{24}{1} = 24 \qquad \frac{-7}{1} = -7$$

- ***Fractions that have a numerator of 0:*** In this case, we have division of 0. The result is 0 (provided the denominator is not 0).

$$\frac{0}{8} = 0 \qquad \frac{0}{56} = 0 \qquad \frac{0}{-11} = 0$$

- ***Fractions that have a denominator of 0:*** In this case, we have division by 0. The division is undefined.

$$\frac{7}{0} \text{ is undefined} \qquad \frac{-18}{0} \text{ is undefined}$$

The Language of Mathematics Perhaps you are wondering about the fraction form $\frac{0}{0}$. It is said to be *undetermined*. This form is important in advanced mathematics courses.

Self Check 3

Simplify, if possible:

a. $\frac{4}{4}$ **b.** $\frac{51}{1}$ **c.** $\frac{45}{0}$ **d.** $\frac{0}{6}$

Now Try **Problem 33**

EXAMPLE 3 Simplify, if possible: **a.** $\frac{12}{12}$ **b.** $\frac{0}{24}$ **c.** $\frac{18}{0}$ **d.** $\frac{9}{1}$

Strategy To simplify each fraction, we will divide the numerator by the denominator, if possible.

WHY A fraction bar indicates division.

Solution

a. $\frac{12}{12} = 1$ This corresponds to dividing a quantity into 12 equal parts, and then considering all 12 of them. We would get 1 whole quantity.

b. $\frac{0}{24} = 0$ This corresponds to dividing a quantity into 24 equal parts, and then considering 0 (none) of them. We would get 0.

c. $\frac{18}{0}$ is undefined This corresponds to dividing a quantity into 0 equal parts, and then considering 18 of them. That is not possible.

d. $\frac{9}{1} = 9$ This corresponds to "dividing" a quantity into 1 equal part, and then considering 9 of them. We would get 9 of those quantities.

The Language of Mathematics Fractions are often referred to as **rational numbers.** All integers are rational numbers, because every integer can be written as a fraction with a denominator of 1. For example,

$$2 = \frac{2}{1}, \quad -5 = \frac{-5}{1}, \quad \text{and} \quad 0 = \frac{0}{1}$$

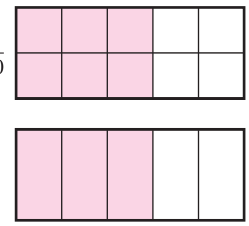
3 Define equivalent fractions.

Fractions can look different but still represent the same part of a whole. To illustrate this, consider the identical rectangular regions on the right. The first one is divided into 10 equal parts. Since 6 of those parts are red, $\frac{6}{10}$ of the figure is shaded.

The second figure is divided into 5 equal parts. Since 3 of those parts are red, $\frac{3}{5}$ of the figure is shaded. We can conclude that $\frac{6}{10} = \frac{3}{5}$ because $\frac{6}{10}$ and $\frac{3}{5}$ represent the same shaded portion of the figure. We say that $\frac{6}{10}$ and $\frac{3}{5}$ are *equivalent fractions*.

$\frac{6}{10}$

$\frac{3}{5}$



Equivalent Fractions

Two fractions are **equivalent** if they represent the same number. **Equivalent fractions** represent the same portion of a whole.

4 Build equivalent fractions.

Writing a fraction as an equivalent fraction with a *larger* denominator is called **building** the fraction. To build a fraction, we use a familiar property from Chapter 1 that is also true for fractions:

Multiplication Property of 1

The product of any fraction and 1 is that fraction.

We also use the following rule for multiplying fractions. (It will be discussed in greater detail in the next section.)

Multiplying Fractions

To multiply two fractions, multiply the numerators and multiply the denominators.

To build an equivalent fraction for $\frac{1}{2}$ with a denominator of 8, we first ask, "What number times 2 equals 8?" To answer that question we *divide* 8 by 2 to get 4. Since we need to multiply the denominator of $\frac{1}{2}$ by 4 to obtain a denominator of 8, it follows that $\frac{4}{4}$ should be the form of 1 that is used to build an equivalent fraction for $\frac{1}{2}$.

$\frac{1}{2} = \frac{1}{2} \cdot \frac{4}{4}$ Multiply $\frac{1}{2}$ by 1 in the form of $\frac{4}{4}$. Note the form of 1 highlighted in red.

$= \frac{1 \cdot 4}{2 \cdot 4}$ Use the rule for multiplying two fractions. Multiply the numerators. Multiply the denominators.

$= \frac{4}{8}$

We have found that $\frac{4}{8}$ is equivalent to $\frac{1}{2}$. To build an equivalent fraction for $\frac{1}{2}$ with a denominator of 8, we *multiplied by a factor equal to 1* in the form of $\frac{4}{4}$. Multiplying $\frac{1}{2}$ by $\frac{4}{4}$ changes its appearance but does not change its value, because we are multiplying it by 1.

Building Fractions

To build a fraction, *multiply it by a factor of 1* in the form $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}$, and so on.

The Language of Mathematics Building an equivalent fraction with a larger denominator is also called *expressing a fraction in higher terms*.

Self Check 4

Write $\frac{5}{8}$ as an equivalent fraction with a denominator of 24.

Now Try **Problems 37 and 49**

EXAMPLE 4 Write $\dfrac{3}{5}$ as an equivalent fraction with a denominator of 35.

Strategy We will compare the given denominator to the required denominator and ask, "What number times 5 equals 35?"

WHY The answer to that question helps us determine the form of 1 to use to build an equivalent fraction.

Solution

To answer the question "What number times 5 equals 35?" we *divide* 35 by 5 to get 7. Since we need to multiply the denominator of $\frac{3}{5}$ by 7 to obtain a denominator of 35, it follows that $\frac{7}{7}$ should be the form of 1 that is used to build an equivalent fraction for $\frac{3}{5}$.

$$\frac{3}{5} = \frac{3}{5} \cdot \frac{7}{7} \qquad \text{Multiply } \tfrac{3}{5} \text{ by a form of 1: } \tfrac{7}{7} = 1.$$

$$= \frac{3 \cdot 7}{5 \cdot 7} \qquad \begin{array}{l}\text{Multiply the numerators.}\\ \text{Multiply the denominators.}\end{array}$$

$$= \frac{21}{35}$$

We have found that $\dfrac{21}{35}$ is equivalent to $\dfrac{3}{5}$.

Success Tip To build an equivalent fraction in Example 4, we multiplied $\dfrac{3}{5}$ by 1 in the form of $\dfrac{7}{7}$. As a result of that step, the numerator and the denominator of $\dfrac{3}{5}$ were multiplied by 7:

$$\frac{3 \cdot 7}{5 \cdot 7} \quad \begin{array}{l}\longleftarrow \text{The numerator is multiplied by 7.}\\ \longleftarrow \text{The denominator is multiplied by 7.}\end{array}$$

This process illustrates the following property of fractions.

The Fundamental Property of Fractions

If the numerator and denominator of a fraction are multiplied by the same nonzero number, the resulting fraction is equivalent to the original fraction.

Since multiplying the numerator and denominator of a fraction by the same nonzero number produces an equivalent fraction, your instructor may allow you to begin your solution to problems like Example 4 as shown in the Success Tip above.

EXAMPLE 5 Write 4 as an equivalent fraction with a denominator of 6.

Strategy We will express 4 as the fraction $\frac{4}{1}$ and build an equivalent fraction by multiplying it by $\frac{6}{6}$.

WHY Since we need to multiply the denominator of $\frac{4}{1}$ by 6 to obtain a denominator of 6, it follows that $\frac{6}{6}$ should be the form of 1 that is used to build an equivalent fraction for $\frac{4}{1}$.

Solution

$$4 = \frac{4}{1} \quad \text{Write 4 as a fraction: } 4 = \frac{4}{1}.$$

$$= \frac{4}{1} \cdot \frac{6}{6} \quad \text{Build an equivalent fraction by multiplying } \frac{4}{1} \text{ by a form of 1: } \frac{6}{6} = 1.$$

$$= \frac{4 \cdot 6}{1 \cdot 6} \quad \text{Multiply the numerators. Multiply the denominators.}$$

$$= \frac{24}{6}$$

Self Check 5

Write 10 as an equivalent fraction with a denominator of 3.

Now Try **Problem 57**

5 Simplify fractions.

Every fraction can be written in infinitely many equivalent forms. For example, some equivalent forms of $\frac{10}{15}$ are:

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18} = \frac{14}{21} = \frac{16}{24} = \frac{18}{27} = \frac{20}{30} = \ldots$$

Of all of the equivalent forms in which we can write a fraction, we often need to determine the one that is in *simplest form.*

Simplest Form of a Fraction

A fraction is in **simplest form,** or **lowest terms,** when the numerator and denominator have no common factors other than 1.

EXAMPLE 6 Are the following fractions in simplest form? **a.** $\frac{12}{27}$ **b.** $\frac{5}{8}$

Strategy We will determine whether the numerator and denominator have any common factors other than 1.

WHY If the numerator and denominator have no common factors other than 1, the fraction is in simplest form.

Solution

a. The factors of the numerator, 12, are: **1**, 2, **3**, 4, 6, 12
The factors of the denominator, 27, are: **1**, **3**, 9, 27

Since the numerator and denominator have a common factor of 3, the fraction $\frac{12}{27}$ is *not* in simplest form.

b. The factors of the numerator, 5, are: **1**, 5
The factors of the denominator, 8, are: **1**, 2, 4, 8

Since the only common factor of the numerator and denominator is 1, the fraction $\frac{5}{8}$ is in simplest form.

Self Check 6

Are the following fractions in simplest form?

a. $\frac{4}{21}$

b. $\frac{6}{20}$

Now Try **Problem 61**

To **simplify a fraction,** we write it in simplest form by *removing a factor equal to 1*. For example, to simplify $\frac{10}{15}$, we note that the greatest factor common to the numerator and denominator is 5 and proceed as follows:

$$\frac{10}{15} = \frac{2 \cdot 5}{3 \cdot 5}$$

Factor 10 and 15. Note the form of 1 highlighted in red.

$$= \frac{2}{3} \cdot \frac{5}{5}$$

Use the rule for multiplying fractions in reverse: write $\frac{2 \cdot 5}{3 \cdot 5}$ as the product of two fractions, $\frac{2}{3}$ and $\frac{5}{5}$.

$$= \frac{2}{3} \cdot 1$$

A number divided by itself is equal to 1: $\frac{5}{5} = 1$.

$$= \frac{2}{3}$$

Use the multiplication property of 1: the product of any fraction and 1 is that fraction.

We have found that the simplified form of $\frac{10}{15}$ is $\frac{2}{3}$. To simplify $\frac{10}{15}$, we *removed a factor equal to 1* in the form of $\frac{5}{5}$. The result, $\frac{2}{3}$, is equivalent to $\frac{10}{15}$.

To streamline the simplifying process, we can replace pairs of factors common to the numerator and denominator with the equivalent fraction $\frac{1}{1}$.

Self Check 7

Simplify each fraction:

a. $\frac{10}{25}$

b. $\frac{3}{9}$

Now Try **Problems 65 and 69**

EXAMPLE 7

Simplify each fraction: **a.** $\frac{6}{10}$ **b.** $\frac{7}{21}$

Strategy We will factor the numerator and denominator. Then we will look for any factors common to the numerator and denominator and remove them.

WHY We need to make sure that the numerator and denominator have no common factors other than 1. If that is the case, then the fraction is in *simplest form.*

Solution

a.

$$\frac{6}{10} = \frac{2 \cdot 3}{2 \cdot 5}$$

To prepare to simplify, factor 6 and 10. Note the form of 1 highlighted in red.

$$= \frac{\overset{1}{\cancel{2}} \cdot 3}{\underset{1}{\cancel{2}} \cdot 5}$$

Simplify by removing the common factor of 2 from the numerator and denominator. A slash / and the 1's are used to show that $\frac{2}{2}$ is replaced by the equivalent fraction $\frac{1}{1}$. A factor equal to 1 in the form of $\frac{2}{2}$ was removed.

$$= \frac{3}{5}$$

Multiply the remaining factors in the numerator: $1 \cdot 3 = 3$. Multiply the remaining factors in the denominator: $1 \cdot 5 = 5$.

Since 3 and 5 have no common factors (other than 1), $\frac{3}{5}$ is in simplest form.

b.

$$\frac{7}{21} = \frac{7}{3 \cdot 7}$$

To prepare to simplify, factor 21.

$$= \frac{\overset{1}{\cancel{7}}}{3 \cdot \underset{1}{\cancel{7}}}$$

Simplify by removing the common factor of 7 from the numerator and denominator.

$$= \frac{1}{3}$$

Multiply the remaining factors in the denominator: $1 \cdot 3 = 3$.

Caution! Don't forget to write the 1's when removing common factors of the numerator and the denominator. Failure to do so can lead to the common mistake shown below.

$$\frac{7}{21} = \frac{\cancel{7}}{3 \cdot \cancel{7}} = \frac{0}{3}$$

We can easily identify common factors of the numerator and the denominator of a fraction if we write them in prime-factored form.

EXAMPLE 8

Simplify each fraction, if possible: **a.** $\frac{90}{105}$ **b.** $\frac{25}{27}$

Strategy We begin by prime factoring the numerator, 90, and denominator, 105. Then we look for any factors common to the numerator and denominator and remove them.

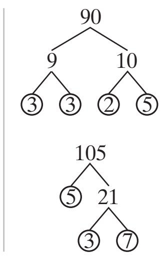
WHY When the numerator and/or denominator of a fraction are large numbers, such as 90 and 105, writing their prime factorizations is helpful in identifying any common factors.

Solution

a. $\frac{90}{105} = \frac{2 \cdot 3 \cdot 3 \cdot 5}{3 \cdot 5 \cdot 7}$ *To prepare to simplify, write 90 and 105 in prime-factored form.*

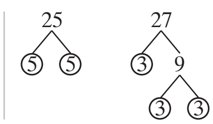
$= \frac{2 \cdot \cancel{3}^{1} \cdot 3 \cdot \cancel{5}^{1}}{\cancel{3}_{1} \cdot \cancel{5}_{1} \cdot 7}$ *Remove the common factors of 3 and 5 from the numerator and denominator. Slashes and 1's are used to show that $\frac{3}{3}$ and $\frac{5}{5}$ are replaced by the equivalent fraction $\frac{1}{1}$. A factor equal to 1 in the form of $\frac{3 \cdot 5}{3 \cdot 5} = \frac{15}{15}$ was removed.*

$= \frac{6}{7}$ *Multiply the remaining factors in the numerator: $2 \cdot 1 \cdot 3 \cdot 1 = 6$. Multiply the remaining factors in the denominator: $1 \cdot 1 \cdot 7 = 7$.*



Since 6 and 7 have no common factors (other than 1), $\frac{6}{7}$ is in simplest form.

b. $\frac{25}{27} = \frac{5 \cdot 5}{3 \cdot 3 \cdot 3}$ *Write 25 and 27 in prime-factored form.*



Since 25 and 27 have no common factors, other than 1, the fraction $\frac{25}{27}$ is in simplest form.

Self Check 8

Simplify each fraction, if possible:

a. $\frac{70}{126}$

b. $\frac{16}{81}$

Now Try **Problems 77 and 81**

EXAMPLE 9

Simplify: $\frac{63}{36}$

Strategy We will prime factor the numerator and denominator. Then we will look for any factors common to the numerator and denominator and remove them.

WHY We need to make sure that the numerator and denominator have no common factors other than 1. If that is the case, then the fraction is in *simplest form.*

Solution

$\frac{63}{36} = \frac{3 \cdot 3 \cdot 7}{2 \cdot 2 \cdot 3 \cdot 3}$ *To prepare to simplify, write 63 and 36 in prime-factored form.*

$= \frac{\cancel{3}^{1} \cdot \cancel{3}^{1} \cdot 7}{2 \cdot 2 \cdot \cancel{3}_{1} \cdot \cancel{3}_{1}}$ *Simplify by removing the common factors of 3 from the numerator and denominator.*

$= \frac{7}{4}$ *Multiply the remaining factors in the numerator: $1 \cdot 1 \cdot 7 = 7$. Multiply the remaining factors in the denominator: $2 \cdot 2 \cdot 1 \cdot 1 = 4$.*

3⌊63	2⌊36
3⌊21	2⌊18
7	3⌊9
	3

Success Tip If you recognized that 63 and 36 have a common factor of 9, you may remove that common factor from the numerator and denominator without writing the prime factorizations. However, make sure that the numerator and denominator of the resulting fraction do not have any common factors. If they do, continue to simplify.

$\frac{63}{36} = \frac{7 \cdot \cancel{9}^{1}}{4 \cdot \cancel{9}_{1}} = \frac{7}{4}$ *Factor 63 as $7 \cdot 9$ and 36 as $4 \cdot 9$, and then remove the common factor of 9 from the numerator and denominator.*

Self Check 9

Simplify: $\frac{162}{72}$

Now Try **Problem 89**

Use the following steps to simplify a fraction.

Simplifying Fractions

To simplify a fraction, *remove factors equal to 1* of the form $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}$, and so on, using the following procedure:

1. Factor (or prime factor) the numerator and denominator to determine their common factors.
2. Remove factors equal to 1 by replacing each pair of factors common to the numerator and denominator with the equivalent fraction $\frac{1}{1}$.
3. Multiply the remaining factors in the numerator and in the denominator.

Negative fractions are simplified in the same way as positive fractions. Just remember to write a negative sign $-$ in front of each step of the solution. For example, to simplify $-\frac{15}{33}$ we proceed as follows:

$$-\frac{15}{33} = -\frac{\overset{1}{\cancel{3}} \cdot 5}{\underset{1}{\cancel{3}} \cdot 11}$$

$$= -\frac{5}{11}$$

ANSWERS TO SELF CHECKS

1. a. numerator: 7; denominator: 9 **b.** numerator: 21; denominator: 20 **2. a.** $\frac{11}{31}$ **b.** $\frac{20}{31}$ **3. a.** 1 **b.** 51 **c.** undefined **d.** 0 **4.** $\frac{15}{24}$ **5.** $\frac{30}{3}$ **6. a.** yes **b.** no **7. a.** $\frac{2}{5}$ **b.** $\frac{1}{3}$ **8. a.** $\frac{5}{9}$ **b.** in simplest form **9.** $\frac{9}{4}$

SECTION 3.1 STUDY SET

VOCABULARY

Fill in the blanks.

1. A ________ describes the number of equal parts of a whole.

2. For the fraction $\frac{7}{8}$, the __________ is 7 and the ____________ is 8.

3. If the numerator of a fraction is less than its denominator, the fraction is called a _______ fraction. If the numerator of a fraction is greater than or equal to its denominator it is called an _________ fraction.

4. Each of the following fractions is a form of ____.

$$\frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \frac{7}{7} = \frac{8}{8} = \frac{9}{9} = \dots$$

5. Two fractions are __________ if they represent the same number.

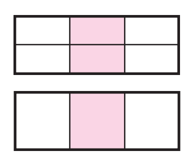
6. __________ fractions represent the same portion of a whole.

7. Writing a fraction as an equivalent fraction with a larger denominator is called ________ the fraction.

8. A fraction is in ________ form, or lowest terms, when the numerator and denominator have no common factors other than 1.

CONCEPTS

9. What concept studied in this section is shown on the right?



10. What concept studied in this section does the following statement illustrate?

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \dots$$

11. Classify each fraction as a proper fraction or an improper fraction.

a. $\dfrac{37}{24}$ **b.** $\dfrac{1}{3}$

c. $\dfrac{71}{100}$ **d.** $\dfrac{9}{9}$

12. Remove the common factors of the numerator and denominator to simplify the fraction:

$$\frac{2\cdot3\cdot3\cdot5}{2\cdot3\cdot5\cdot7}$$

13. What common factor (other than 1) do the numerator and the denominator of the fraction $\frac{10}{15}$ have?

Fill in the blank.

14. Multiplication property of 1: The product of any fraction and 1 is that ________.

15. Multiplying fractions: To multiply two fractions, multiply the __________ and multiply the denominators.

16. a. Consider the following solution: $\dfrac{2}{3}=\dfrac{2}{3}\cdot\boxed{}$

$$=\frac{8}{12}$$

To build an equivalent fraction for $\frac{2}{3}$ with a denominator of 12, ________ it by a factor equal to 1 in the form of $\boxed{}$.

b. Consider the following solution: $\dfrac{15}{27}=\dfrac{\overset{1}{\cancel{3}}\cdot5}{\underset{1}{\cancel{3}}\cdot9}$

$$=\frac{5}{9}$$

To simplify the fraction $\frac{15}{27}$, ________ a factor equal to 1 of the form $\boxed{}$.

NOTATION

17. Write the fraction $\dfrac{7}{-8}$ in two other ways.

18. Write each integer as a fraction.

a. 8 **b.** –25

Complete each solution.

19. Build an equivalent fraction for $\dfrac{1}{6}$ with a denominator of 18.

$$\frac{1}{6}=\frac{1}{6}\cdot\frac{3}{\boxed{}}$$

$$=\frac{\boxed{}\cdot3}{6\cdot\boxed{}}$$

$$=\frac{3}{\boxed{}}$$

20. Simplify: $\dfrac{18}{24}$

$$\frac{18}{24}=\frac{2\cdot\boxed{}\cdot3}{2\cdot2\cdot2\cdot\boxed{}}$$

$$=\frac{\overset{1}{\cancel{\boxed{}}}\cdot3\cdot\overset{1}{\cancel{\boxed{}}}}{\underset{1}{\cancel{2}}\cdot2\cdot2\cdot\underset{1}{\cancel{\boxed{}}}}$$

$$=\frac{3}{\boxed{}}$$

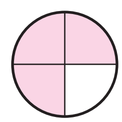
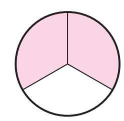
GUIDED PRACTICE

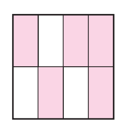
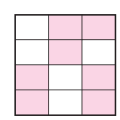
Identify the numerator and denominator of each fraction. See Example 1.

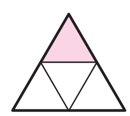
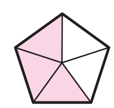
21. $\dfrac{4}{5}$ **22.** $\dfrac{7}{8}$

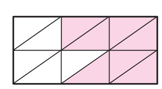
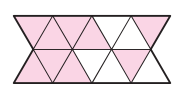
23. $\dfrac{17}{10}$ **24.** $\dfrac{29}{21}$

Write a fraction to describe what part of the figure is shaded. Write a fraction to describe what part of the figure is not shaded. See Example 2.

25.  **26.** 

27.  **28.** 

29.  **30.** 

31.  **32.** 

Simplify, if possible. See Example 3.

33. a. $\dfrac{4}{1}$ **b.** $\dfrac{8}{8}$

c. $\dfrac{0}{12}$ **d.** $\dfrac{1}{0}$

34. a. $\dfrac{25}{1}$ **b.** $\dfrac{14}{14}$

c. $\dfrac{0}{1}$ **d.** $\dfrac{83}{0}$

35. a. $\dfrac{5}{0}$ **b.** $\dfrac{0}{50}$

c. $\dfrac{33}{33}$ **d.** $\dfrac{75}{1}$

36. **a.** $\frac{0}{64}$ **b.** $\frac{27}{0}$

c. $\frac{125}{125}$ **d.** $\frac{98}{1}$

Write each fraction as an equivalent fraction with the indicated denominator. See Example 4.

37. $\frac{7}{8}$, denominator 40

38. $\frac{3}{4}$, denominator 24

39. $\frac{4}{9}$, denominator 27

40. $\frac{5}{7}$, denominator 49

41. $\frac{5}{6}$, denominator 54

42. $\frac{2}{3}$, denominator 27

43. $\frac{2}{7}$, denominator 14

44. $\frac{3}{10}$, denominator 50

45. $\frac{1}{2}$, denominator 30

46. $\frac{1}{3}$, denominator 60

47. $\frac{11}{16}$, denominator 32

48. $\frac{9}{10}$, denominator 60

49. $\frac{5}{4}$, denominator 28

50. $\frac{9}{4}$, denominator 44

51. $\frac{16}{15}$, denominator 45

52. $\frac{13}{12}$, denominator 36

Write each whole number as an equivalent fraction with the indicated denominator. See Example 5.

53. 4, denominator 9

54. 4, denominator 3

55. 6, denominator 8

56. 3, denominator 6

57. 3, denominator 5

58. 7, denominator 4

59. 14, denominator 2

60. 10, denominator 9

Are the following fractions in simplest form? See Example 6.

61. **a.** $\frac{12}{16}$ **b.** $\frac{3}{25}$

62. **a.** $\frac{9}{24}$ **b.** $\frac{7}{36}$

63. **a.** $\frac{35}{36}$ **b.** $\frac{18}{21}$

64. **a.** $\frac{22}{45}$ **b.** $\frac{21}{56}$

Simplify each fraction, if possible. See Example 7.

65. $\frac{6}{9}$

66. $\frac{15}{20}$

67. $\frac{16}{20}$

68. $\frac{25}{35}$

69. $\frac{5}{15}$

70. $\frac{6}{30}$

71. $\frac{2}{48}$

72. $\frac{2}{42}$

Simplify each fraction, if possible. See Example 8.

73. $\frac{36}{96}$

74. $\frac{48}{120}$

75. $\frac{16}{17}$

76. $\frac{14}{25}$

77. $\frac{55}{62}$

78. $\frac{41}{51}$

79. $\frac{50}{55}$

80. $\frac{22}{88}$

81. $\frac{60}{108}$

82. $\frac{75}{275}$

83. $\frac{180}{210}$

84. $\frac{90}{120}$

Simplify each fraction. See Example 9.

85. $\frac{306}{234}$

86. $\frac{208}{117}$

87. $\frac{15}{6}$

88. $\frac{24}{16}$

89. $\frac{420}{144}$

90. $\frac{216}{189}$

91. $-\frac{4}{68}$

92. $-\frac{3}{42}$

93. $-\frac{90}{105}$

94. $-\frac{98}{126}$

95. $-\frac{16}{26}$

96. $-\frac{81}{132}$

TRY IT YOURSELF

Tell whether each pair of fractions are equivalent by simplifying each fraction.

97. $\frac{2}{14}$ and $\frac{6}{36}$

98. $\frac{3}{12}$ and $\frac{4}{24}$

99. $\frac{22}{34}$ and $\frac{33}{51}$

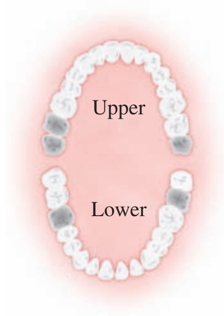
100. $\frac{4}{30}$ and $\frac{12}{90}$

APPLICATIONS

101. DENTISTRY Refer to the dental chart.

a. How many teeth are shown on the chart?

b. What fraction of this set of teeth have fillings?



102. TIME CLOCKS For each clock, what fraction of the hour has passed? Write your answers in simplified form. (*Hint:* There are 60 minutes in an hour.)

a.


b.

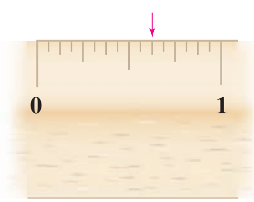

c.


d.

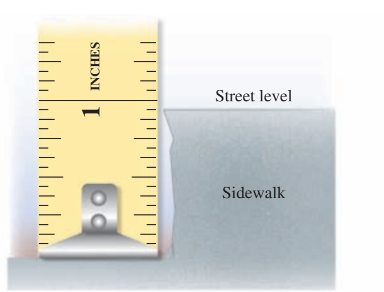

103. RULERS The illustration below shows a ruler.

a. How many spaces are there between the numbers 0 and 1?

b. To what fraction is the arrow pointing? Write your answer in simplified form.



104. SINKHOLES The illustration below shows a side view of a drop in the sidewalk near a sinkhole. Describe the movement of the sidewalk using a signed fraction.

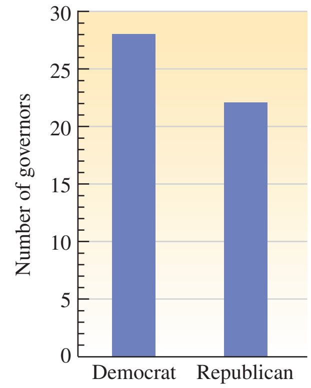


105. POLITICAL PARTIES The graph shows the number of Democrat and Republican governors of the 50 states, as of February 1, 2009.

a. How many Democrat governors are there? How many Republican governors are there?

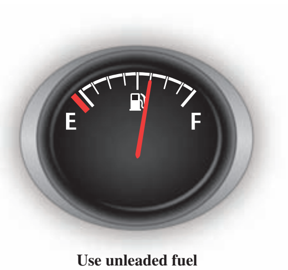
b. What fraction of the governors are Democrats? Write your answer in simplified form.

c. What fraction of the governors are Republicans? Write your answer in simplified form.



Source: thegreenpapers.com

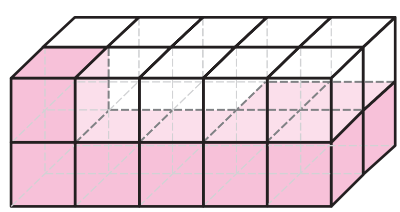
106. GAS TANKS Write fractions to describe the amount of gas left in the tank and the amount of gas that has been used.



107. SELLING CONDOS The model below shows a new condominium development. The condos that have been sold are shaded.

a. How many units are there in the development?

b. What fraction of the units in the development have been sold? What fraction have not been sold? Write your answers in simplified form.

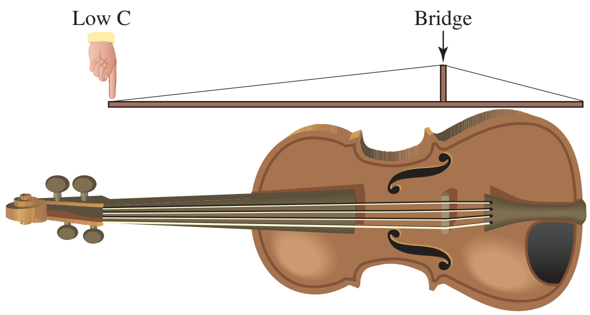


108. MUSIC The illustration shows a side view of the finger position needed to produce a length of string (from the bridge to the fingertip) that gives low C on a violin. To play other notes, fractions of that length are used. Locate these finger positions on the illustration.

a. $\frac{1}{2}$ of the length gives middle C.

b. $\frac{3}{4}$ of the length gives F above low C.

c. $\frac{2}{3}$ of the length gives G.



109. MEDICAL CENTERS Hospital designers have located a nurse's station at the center of a circular building. Show how to divide the surrounding office space (shaded in grey) so that each medical department has the fractional amount assigned to it. Label each department.

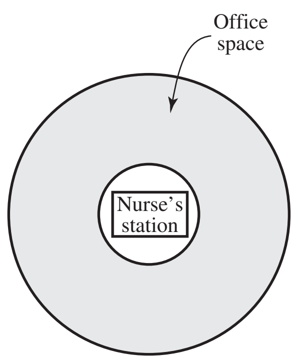
$\frac{2}{12}$: Radiology

$\frac{5}{12}$: Pediatrics

$\frac{1}{12}$: Laboratory

$\frac{3}{12}$: Orthopedics

$\frac{1}{12}$: Pharmacy



Medical Center

110. GDP The gross domestic product (GDP) is the official measure of the size of the U.S. economy. It represents the market value of all goods and services that have been bought during a given period of time. The GDP for the second quarter of 2008 is listed below. What is meant by the phrase *second quarter of 2008*?

Second quarter of 2008 $14,294,500,000,000

Source: *The World Almanac and Book of Facts,* 2009

WRITING

111. Explain the concept of equivalent fractions. Give an example.

112. What does it mean for a fraction to be in simplest form? Give an example.

113. Why can't we say that $\frac{2}{5}$ of the figure below is shaded?

114. Perhaps you have heard the following joke:

A pizza parlor waitress asks a customer if he wants the pizza cut into four pieces or six pieces or eight pieces. The customer then declares that he wants either four or six pieces of pizza "because I can't eat eight."

Explain what is wrong with the customer's thinking.

115. **a.** What type of problem is shown below? Explain the solution.

$$\frac{1}{2} = \frac{1}{2} \cdot \frac{4}{4} = \frac{4}{8}$$

b. What type of problem is shown below? Explain the solution.

$$\frac{15}{35} = \frac{3 \cdot \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \cdot 7} = \frac{3}{7}$$

116. Explain the difference in the two approaches used to simplify $\frac{20}{28}$. Are the results the same?

$$\frac{\overset{1}{\cancel{4}} \cdot 5}{\underset{1}{\cancel{4}} \cdot 7} \quad \text{and} \quad \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 5}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 7}$$

REVIEW

117. PAYCHECKS *Gross pay* is what a worker makes before deductions and *net pay* is what is left after taxes, health benefits, union dues, and other deductions are taken out. Suppose a worker's monthly gross pay is $3,575. If deductions of $235, $782, $148, and $103 are taken out of his check, what is his monthly net pay?

118. HORSE RACING One day, a man bet on all eight horse races at Santa Anita Racetrack. He won $168 on the first race and he won $105 on the fourth race. He lost his $50-bets on each of the other races. Overall, did he win or lose money betting on the horses? How much?

SECTION 3.2

Multiplying Fractions

Objectives

1. Multiply fractions.
2. Simplify answers when multiplying fractions.
3. Evaluate exponential expressions that have fractional bases.
4. Solve application problems by multiplying fractions.
5. Find the area of a triangle.

In the next three sections, we discuss how to add, subtract, multiply, and divide fractions. We begin with the operation of multiplication.

1 Multiply fractions.

To develop a rule for multiplying fractions, let's consider a real-life application.

Suppose $\frac{3}{5}$ of the last page of a school newspaper is devoted to campus sports coverage. To show this, we can divide the page into fifths, and shade 3 of them red.

Furthermore, suppose that $\frac{1}{2}$ of the sports coverage is about women's teams. We can show that portion of the page by dividing the already colored region into two halves, and shading one of them in purple.

To find the fraction represented by the purple shaded region, the page needs to be divided into equal-size parts. If we extend the dashed line downward, we see there are 10 equal-sized parts. The purple shaded parts are 3 out of 10, or $\frac{3}{10}$, of the page. Thus, $\frac{3}{10}$ of the last page of the school newspaper is devoted to women's sports.

In this example, we have found that

$$\frac{1}{2} \text{ of } \frac{3}{5} \text{ is } \frac{3}{10}$$

$$\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$$

Since the key word *of* indicates multiplication, and the key word *is* means equals, we can translate this statement to symbols.

Two observations can be made from this result.

- The numerator of the answer is the product of the numerators of the original fractions.

$1 \cdot 3 = 3$

$$\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10} \quad \text{Answer}$$

$2 \cdot 5 = 10$

- The denominator of the answer is the product of the denominators of the original fractions.

These observations illustrate the following rule for multiplying two fractions.

Multiplying Fractions

To multiply two fractions, multiply the numerators and multiply the denominators. Simplify the result, if possible.

Success Tip In the newspaper example, we found a *part of a part* of a page. Multiplying proper fractions can be thought of in this way. When taking a *part of a part* of something, the result is always smaller than the original part that you began with.

EXAMPLE 1

Multiply: **a.** $\frac{1}{6} \cdot \frac{1}{4}$ **b.** $\frac{7}{8} \cdot \frac{3}{5}$

Strategy We will multiply the numerators and denominators, and make sure that the result is in simplest form.

WHY This is the rule for multiplying two fractions.

Solution

a. $\frac{1}{6} \cdot \frac{1}{4} = \frac{1 \cdot 1}{6 \cdot 4}$ Multiply the numerators. Multiply the denominators.

$= \frac{1}{24}$ Since 1 and 24 have no common factors other than 1, the result is in simplest form.

b. $\frac{7}{8} \cdot \frac{3}{5} = \frac{7 \cdot 3}{8 \cdot 5}$ Multiply the numerators. Multiply the denominators.

$= \frac{21}{40}$ Since 21 and 40 have no common factors other than 1, the result is in simplest form.

Self Check 1

Multiply:

a. $\frac{1}{2} \cdot \frac{1}{8}$

b. $\frac{5}{9} \cdot \frac{2}{3}$

Now Try **Problems 17 and 21**

The sign rules for multiplying integers also hold for multiplying fractions. When we multiply two fractions with *like* signs, the product is positive. When we multiply two fractions with *unlike* signs, the product is negative.

EXAMPLE 2 Multiply: $-\frac{3}{4}\left(\frac{1}{8}\right)$

Strategy We will use the rule for multiplying two fractions that have different (unlike) signs.

WHY One fraction is positive and one is negative.

Solution

$$-\frac{3}{4}\left(\frac{1}{8}\right) = -\frac{3 \cdot 1}{4 \cdot 8}$$

Multiply the numerators.
Multiply the denominators.
Since the fractions have unlike signs, make the answer negative.

$$= -\frac{3}{32}$$

Since 3 and 32 have no common factors other than 1, the result is in simplest form.

Self Check 2

Multiply: $\frac{5}{6}\left(-\frac{1}{3}\right)$

Now Try **Problem 25**

EXAMPLE 3 Multiply: $\frac{1}{2} \cdot 3$

Strategy We will begin by writing the integer 3 as a fraction.

WHY Then we can use the rule for multiplying two fractions to find the product.

Solution

$$\frac{1}{2} \cdot 3 = \frac{1}{2} \cdot \frac{3}{1}$$

Write 3 as a fraction: $3 = \frac{3}{1}$.

$$= \frac{1 \cdot 3}{2 \cdot 1}$$

Multiply the numerators.
Multiply the denominators.

$$= \frac{3}{2}$$

Since 3 and 2 have no common factors other than 1, the result is in simplest form.

Self Check 3

Multiply: $\frac{1}{3} \cdot 7$

Now Try **Problem 29**

2 Simplify answers when multiplying fractions.

After multiplying two fractions, we need to simplify the result, if possible. To do that, we can use the procedure discussed in Section 3.1 by removing pairs of common factors of the numerator and denominator.

EXAMPLE 4 Multiply and simplify: $\frac{5}{8} \cdot \frac{4}{5}$

Strategy We will multiply the numerators and denominators, and make sure that the result is in simplest form.

WHY This is the rule for multiplying two fractions.

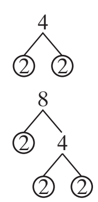
Solution

$$\frac{5}{8} \cdot \frac{4}{5} = \frac{5 \cdot 4}{8 \cdot 5}$$

Multiply the numerators.
Multiply the denominators.

$$= \frac{5 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 5}$$

To prepare to simplify, write 4 and 8 in prime-factored form.



$$= \frac{\overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot \underset{1}{\cancel{5}}}$$

To simplify, remove the common factors of 2 and 5 from the numerator and denominator.

$$= \frac{1}{2}$$

Multiply the remaining factors in the numerator: $1 \cdot 1 \cdot 1 = 1$.
Multiple the remaining factors in the denominator: $1 \cdot 1 \cdot 2 \cdot 1 = 2$.

Self Check 4

Multiply and simplify: $\frac{11}{25} \cdot \frac{10}{11}$

Now Try **Problem 33**

Success Tip If you recognized that 4 and 8 have a common factor of 4, you may remove that common factor from the numerator and denominator of the product without writing the prime factorizations. However, make sure that the numerator and denominator of the resulting fraction do not have any common factors. If they do, continue to simplify.

$$\frac{5}{8} \cdot \frac{4}{5} = \frac{5 \cdot 4}{8 \cdot 5} = \frac{\overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{4}}}{2 \cdot \underset{1}{\cancel{4}} \cdot \underset{1}{\cancel{5}}} = \frac{1}{2}$$

Factor 8 as $2 \cdot 4$, and then remove the common factors of 4 and 5 in the numerator and denominator.

The rule for multiplying two fractions can be extended to find the product of three or more fractions.

Self Check 5

Multiply and simplify:

$$\frac{2}{5}\left(-\frac{15}{22}\right)\left(-\frac{11}{26}\right)$$

Now Try **Problem 37**

EXAMPLE 5 Multiply and simplify: $\frac{2}{3}\left(-\frac{9}{14}\right)\left(-\frac{7}{10}\right)$

Strategy We will multiply the numerators and denominators, and make sure that the result is in simplest form.

WHY This is the rule for multiplying three (or more) fractions.

Solution Recall from Section 2.4 that a product is positive when there are an even number of negative factors. Since $\frac{2}{3}\left(-\frac{9}{14}\right)\left(-\frac{7}{10}\right)$ has *two* negative factors, the product is positive.

$$\frac{2}{3}\left(-\frac{9}{14}\right)\left(-\frac{7}{10}\right) = \frac{2}{3}\left(\frac{9}{14}\right)\left(\frac{7}{10}\right)$$

Since the answer is positive, drop both − signs and continue.

$$= \frac{2 \cdot 9 \cdot 7}{3 \cdot 14 \cdot 10}$$

Multiply the numerators. Multiply the denominators.

$$= \frac{2 \cdot 3 \cdot 3 \cdot 7}{3 \cdot 2 \cdot 7 \cdot 2 \cdot 5}$$

To prepare to simplify, write 9, 14, and 10 in prime-factored form.

$$= \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot 3 \cdot \overset{1}{\cancel{7}}}{\underset{1}{\cancel{3}} \cdot 2 \cdot \underset{1}{\cancel{7}} \cdot \underset{1}{\cancel{2}} \cdot 5}$$

To simplify, remove the common factors of 2, 3, and 7 from the numerator and denominator.

$$= \frac{3}{10}$$

Multiply the remaining factors in the numerator. Multiply the remaining factors in the denominator.

Caution! In Example 5, it was very helpful to prime factor and simplify when we did (the third step of the solution). If, instead, you find the product of the numerators and the product of the denominators, the resulting fraction is difficult to simplify because the numerator, 126, and the denominator, 420, are large.

$$\frac{2}{3} \cdot \frac{9}{14} \cdot \frac{7}{10} = \frac{2 \cdot 9 \cdot 7}{3 \cdot 14 \cdot 10} = \frac{126}{420}$$

Factor and simplify at this stage, before multiplying in the numerator and denominator.

Don't multiply in the numerator and denominator and then try to simplify the result. You will get the same answer, but it takes much more work.

3 Evaluate exponential expressions that have fractional bases.

We have evaluated exponential expressions that have whole-number bases and integer bases. If the base of an exponential expression is a fraction, the exponent tells us how many times to write that fraction as a factor. For example,

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3} \cdot \frac{2}{3} = \frac{2 \cdot 2}{3 \cdot 3} = \frac{4}{9}$$

Since the exponent is 2, write the base, $\frac{2}{3}$, as a factor 2 times.

EXAMPLE 6 Evaluate each expression: **a.** $\left(\frac{1}{4}\right)^3$ **b.** $\left(-\frac{2}{3}\right)^2$ **c.** $-\left(\frac{2}{3}\right)^2$

Strategy We will write each exponential expression as a product of repeated factors, and then perform the multiplication. This requires that we identify the base and the exponent.

WHY The exponent tells the number of times the base is to be written as a factor.

Solution

Recall that exponents are used to represent repeated multiplication.

a. We read $\left(\frac{1}{4}\right)^3$ as "one-fourth raised to the third power," or as "one-fourth, cubed."

$$\left(\frac{1}{4}\right)^3 = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$$ Since the exponent is 3, write the base, $\frac{1}{4}$, as a factor 3 times.

$$= \frac{1 \cdot 1 \cdot 1}{4 \cdot 4 \cdot 4}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{1}{64}$$

b. We read $\left(-\frac{2}{3}\right)^2$ as "negative two-thirds raised to the second power," or as "negative two-thirds, squared."

$$\left(-\frac{2}{3}\right)^2 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)$$ Since the exponent is 2, write the base, $-\frac{2}{3}$, as a factor 2 times.

$$= \frac{2 \cdot 2}{3 \cdot 3}$$ The product of two fractions with like signs is positive: Drop the $-$ signs. Multiply the numerators. Multiply the denominators.

$$= \frac{4}{9}$$

c. We read $-\left(\frac{2}{3}\right)^2$ as "the opposite of two-thirds squared." Recall that if the $-$ symbol is not within the parantheses, it is not part of the base.

$$-\left(\frac{2}{3}\right)^2 = -\frac{2}{3} \cdot \frac{2}{3}$$ Since the exponent is 2, write the base, $\frac{2}{3}$, as a factor 2 times.

$$= -\frac{2 \cdot 2}{3 \cdot 3}$$ Multiply the numerators. Multiply the denominators.

$$= -\frac{4}{9}$$

Self Check 6

Evaluate each expression:

a. $\left(\frac{2}{5}\right)^3$

b. $\left(-\frac{3}{4}\right)^2$

c. $-\left(\frac{3}{4}\right)^2$

Now Try **Problem 43**

4 Solve application problems by multiplying fractions.

The key word *of* often appears in application problems involving fractions. When a fraction is followed by the word *of*, such as $\frac{1}{2}$ *of* or $\frac{3}{4}$ *of*, it indicates that we are to find a part of some quantity using multiplication.

EXAMPLE 7 ***How a Bill Becomes Law*** If the President vetoes (refuses to sign) a bill, it takes $\frac{2}{3}$ of those voting in the House of Representatives (and the Senate) to override the veto for it to become law. If all 435 members of the House cast a vote, how many of their votes does it take to override a presidential veto?

Analyze

- It takes $\frac{2}{3}$ *of* those voting to override a veto. *Given*
- All 435 members of the House cast a vote. *Given*
- How many votes does it take to override a Presidential veto? *Find*

Self Check 7

HOW A BILL BECOMES LAW If only 96 Senators are present and cast a vote, how many of their votes does it takes to override a Presidential veto?

Now Try **Problems 45 and 87**

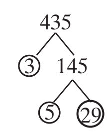
Form The key phrase $\frac{2}{3}$ *of* suggests that we are to find a part of the 435 possible votes using multiplication.

We translate the words of the problem to numbers and symbols.

The number of votes needed in the House to override a veto	is equal to	$\frac{2}{3}$	of	the number of House members that vote.
The number of votes needed in the House to override a veto	$=$	$\frac{2}{3}$	$\cdot$	435

Solve To find the product, we will express 435 as a fraction and then use the rule for multiplying two fractions.

$$\frac{2}{3} \cdot 435 = \frac{2}{3} \cdot \frac{435}{1}$$ Write 435 as a fraction: $435 = \frac{435}{1}$.



$$= \frac{2 \cdot 435}{3 \cdot 1}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{2 \cdot 3 \cdot 5 \cdot 29}{3 \cdot 1}$$ To prepare to simplify, write 435 in prime-factored form: $3 \cdot 5 \cdot 29$.

$$= \frac{2 \cdot \overset{1}{\cancel{3}} \cdot 5 \cdot 29}{\underset{1}{\cancel{3}} \cdot 1}$$ Remove the common factor of 3 from the numerator and denominator.

$$= \frac{290}{1}$$ Multiply the remaining factors in the numerator: $2 \cdot 1 \cdot 5 \cdot 29 = 290$. Multiply the remaining factors in the denominator: $1 \cdot 1 = 1$.

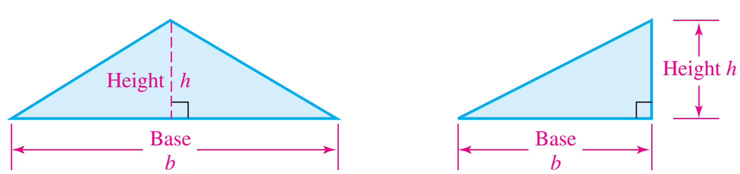
$$= 290$$ Any whole number divided by 1 is equal to that number.

State It would take 290 votes in the House to override a veto.

Check We can estimate to check the result. We will use 440 to approximate the number of House members voting. Since $\frac{1}{2}$ of 440 is 220, and since $\frac{2}{3}$ is a greater part than $\frac{1}{2}$, we would expect the number of votes needed to be *more than* 220. The result of 290 seems reasonable.

5 Find the area of a triangle.

As the figures below show, a triangle has three sides. The length of the base of the triangle can be represented by the letter b and the height by the letter h. The height of a triangle is always perpendicular (makes a square corner) to the base. This is shown by using the symbol ⌐.



Recall that the area of a figure is the amount of surface that it encloses. The area of a triangle can be found by using the following formula.

Area of a Triangle

The area A of a triangle is one-half the product of its base b and its height h.

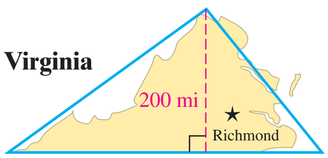
$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) \qquad \text{or} \qquad A = \frac{1}{2} \cdot b \cdot h$$

The Language of Mathematics The formula $A = \frac{1}{2} \cdot b \cdot h$ can be written more simply as $A = \frac{1}{2}bh$. The formula for the area of a triangle can also be written as $A = \frac{bh}{2}$.

EXAMPLE 8 ***Geography*** Approximate the area of the state of Virginia (in square miles) using the triangle shown below.

Strategy We will find the product of $\frac{1}{2}$, 405, and 200.

WHY The formula for the area of a triangle is $A = \frac{1}{2}$(base)(height).



Solution

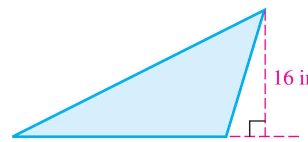
$$\begin{aligned}
A &= \frac{1}{2}bh && \text{This is the formula for the area of a triangle.} \\
&= \frac{1}{2} \cdot 405 \cdot 200 && \tfrac{1}{2}bh \text{ means } \tfrac{1}{2} \cdot b \cdot h. \text{ Substitute 405 for } b \text{ and 200 for } h. \\
&= \frac{1}{2} \cdot \frac{405}{1} \cdot \frac{200}{1} && \text{Write 405 and 200 as fractions.} \\
&= \frac{1 \cdot 405 \cdot 200}{2 \cdot 1 \cdot 1} && \text{Multiply the numerators. Multiply the denominators.} \\
&= \frac{1 \cdot 405 \cdot \overset{1}{\cancel{2}} \cdot 100}{\underset{1}{\cancel{2}} \cdot 1 \cdot 1} && \text{Factor 200 as } 2 \cdot 100. \text{ Then remove the common factor of 2 from the numerator and denominator.} \\
&= 40{,}500 && \text{In the numerator, multiply: } 405 \cdot 100 = 40{,}500.
\end{aligned}$$

The area of the state of Virginia is approximately 40,500 square miles. This can be written as 40,500 mi^2.

Caution! Remember that area is measured in square units, such as in.^{2}, ft^2, and cm^2. Don't forget to write the units in your answer when finding the area of a figure.

Self Check 8

Find the area of the triangle shown below.



Now Try **Problems 49 and 99**

ANSWERS TO SELF CHECKS

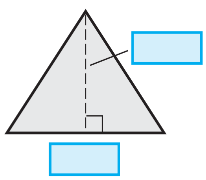
1. a. $\frac{1}{16}$ **b.** $\frac{10}{27}$ **2.** $-\frac{5}{18}$ **3.** $\frac{7}{3}$ **4.** $\frac{2}{5}$ **5.** $\frac{3}{26}$ **6. a.** $\frac{8}{125}$ **b.** $\frac{9}{16}$ **c.** $-\frac{9}{16}$
7. 64 votes **8.** 216 in.^{2}

SECTION 3.2 STUDY SET

VOCABULARY

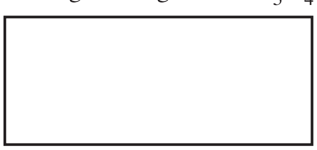
Fill in the blanks.

1. When a fraction is followed by the word *of*, such as $\frac{1}{3}$ *of*, it indicates that we are to find a part of some quantity using ________.
2. The answer to a multiplication is called the ________.
3. To ________ a fraction, we remove common factors of the numerator and denominator.
4. In the expression $\left(\frac{1}{4}\right)^3$, the ______ is $\frac{1}{4}$ and the ________ is 3.
5. The ______ of a triangle is the amount of surface that it encloses.
6. Label the *base* and the *height* of the triangle shown below.



CONCEPTS

7. Fill in the blanks: To multiply two fractions, multiply the ___________ and multiply the ______________. Then ________, if possible.
8. Use the following rectangle to find $\frac{1}{3} \cdot \frac{1}{4}$.



   a. Draw three vertical lines that divide the given rectangle into four equal parts and lightly shade one part. What fractional part of the rectangle did you shade?

   b. To find $\frac{1}{3}$ of the shaded portion, draw two horizontal lines to divide the given rectangle into three equal parts and lightly shade one part. Into how many equal parts is the rectangle now divided? How many parts have been shaded twice?

   c. What is $\frac{1}{3} \cdot \frac{1}{4}$?

9. Determine whether each product is positive or negative. *You do not have to find the answer.*

   a. $-\frac{1}{8} \cdot \frac{3}{5}$ **b.** $-\frac{7}{16}\left(-\frac{2}{21}\right)$

   c. $-\frac{4}{5}\left(\frac{1}{3}\right)\left(-\frac{1}{8}\right)$ **d.** $-\frac{3}{4}\left(-\frac{8}{9}\right)\left(-\frac{1}{2}\right)$

10. Translate each phrase to symbols. *You do not have to find the answer.*

   a. $\frac{7}{10}$ of $\frac{4}{9}$ **b.** $\frac{1}{5}$ of 40

11. Fill in the blanks: Area of a triangle = $\frac{1}{2}$(_____)(______) or A = ____
12. Fill in the blank: Area is measured in _______ units, such as in.^2 and ft^2.

NOTATION

13. Write each of the following integers as a fraction.

   a. 4 **b.** –3

14. Fill in the blanks: $\left(\frac{1}{2}\right)^2$ represents the repeated multiplication $\square \cdot \square$.

Fill in the blanks to complete each solution.

15.
$$\frac{5}{8} \cdot \frac{7}{15} = \frac{5 \cdot \square}{8 \cdot \square}$$
$$= \frac{5 \cdot 7}{\square \cdot 2 \cdot 2 \cdot \square \cdot 5}$$
$$= \frac{\overset{1}{\cancel{\square}} \cdot 7}{2 \cdot 2 \cdot 2 \cdot 3 \cdot \underset{1}{\cancel{\square}}}$$
$$= \frac{7}{\square}$$

16.
$$\frac{7}{12} \cdot \frac{4}{21} = \frac{7 \cdot 4}{\square \cdot \square}$$
$$= \frac{7 \cdot 4}{\square \cdot 4 \cdot 3 \cdot \square}$$
$$= \frac{\overset{1}{\cancel{\square}} \cdot \overset{1}{\cancel{4}}}{3 \cdot \underset{1}{\cancel{4}} \cdot 3 \cdot \underset{1}{\cancel{\square}}}$$
$$= \frac{\square}{9}$$

GUIDED PRACTICE

Multiply. Write the product in simplest form. See Example 1.

17. $\frac{1}{4} \cdot \frac{1}{2}$
18. $\frac{1}{3} \cdot \frac{1}{5}$
19. $\frac{1}{9} \cdot \frac{1}{5}$
20. $\frac{1}{2} \cdot \frac{1}{8}$

21. $\frac{2}{3} \cdot \frac{7}{9}$

22. $\frac{3}{4} \cdot \frac{5}{7}$

23. $\frac{8}{11} \cdot \frac{3}{7}$

24. $\frac{11}{13} \cdot \frac{2}{3}$

Multiply. See Example 2.

25. $-\frac{4}{5} \cdot \frac{1}{3}$

26. $-\frac{7}{9} \cdot \frac{1}{4}$

27. $\frac{5}{6}\left(-\frac{7}{12}\right)$

28. $\frac{2}{15}\left(-\frac{4}{3}\right)$

Multiply. See Example 3.

29. $\frac{1}{8} \cdot 9$

30. $\frac{1}{6} \cdot 11$

31. $\frac{1}{2} \cdot 5$

32. $\frac{1}{2} \cdot 21$

Multiply. Write the product in simplest form. See Example 4.

33. $\frac{11}{10} \cdot \frac{5}{11}$

34. $\frac{5}{4} \cdot \frac{2}{5}$

35. $\frac{6}{49} \cdot \frac{7}{6}$

36. $\frac{13}{4} \cdot \frac{4}{39}$

Multiply. Write the product in simplest form. See Example 5.

37. $\frac{3}{4}\left(-\frac{8}{35}\right)\left(-\frac{7}{12}\right)$

38. $\frac{9}{10}\left(-\frac{4}{15}\right)\left(-\frac{5}{18}\right)$

39. $-\frac{5}{8}\left(\frac{16}{27}\right)\left(-\frac{9}{25}\right)$

40. $-\frac{15}{28}\left(\frac{7}{9}\right)\left(-\frac{18}{35}\right)$

Evaluate each expression. See Example 6.

41. a. $\left(\frac{3}{5}\right)^2$ **b.** $\left(-\frac{3}{5}\right)^2$

42. a. $\left(\frac{4}{9}\right)^2$ **b.** $\left(-\frac{4}{9}\right)^2$

43. a. $-\left(-\frac{1}{6}\right)^2$ **b.** $\left(-\frac{1}{6}\right)^3$

44. a. $-\left(-\frac{2}{5}\right)^2$ **b.** $\left(-\frac{2}{5}\right)^3$

Find each product. Write your answer in simplest form. See Example 7.

45. $\frac{3}{4}$ of $\frac{5}{8}$

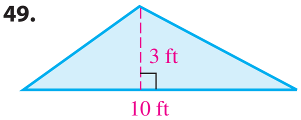
46. $\frac{4}{5}$ of $\frac{3}{7}$

47. $\frac{1}{6}$ of 54

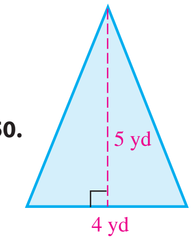
48. $\frac{1}{9}$ of 36

Find the area of each triangle. See Example 8.

49.



50.



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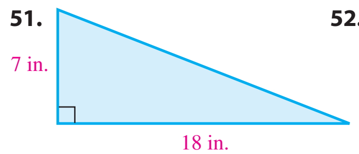
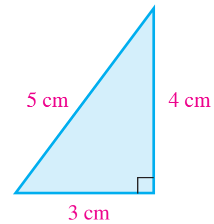


52.



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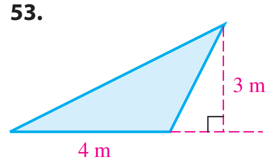
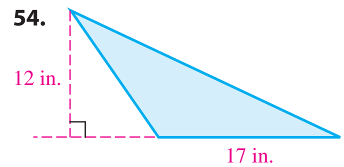
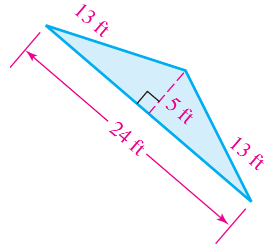


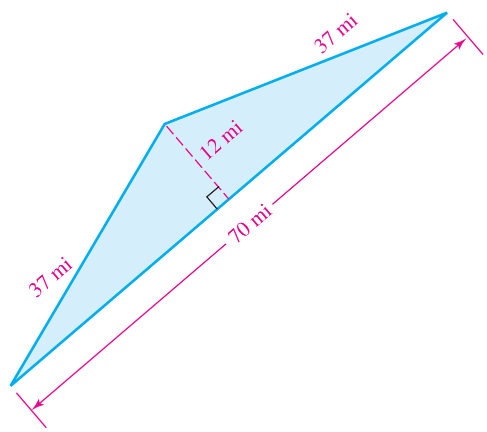
54.



55.



56.



TRY IT YOURSELF

57. Complete the multiplication table of fractions.

$\cdot$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$
$\frac{1}{2}$					
$\frac{1}{3}$					
$\frac{1}{4}$					
$\frac{1}{5}$					
$\frac{1}{6}$					

58. Complete the table by finding the original fraction, given its square.

Original fraction squared	Original fraction
$\frac{1}{9}$	
$\frac{1}{100}$	
$\frac{4}{25}$	
$\frac{16}{49}$	
$\frac{81}{36}$	
$\frac{9}{121}$	

Multiply. Write the product in simplest form.

59. $-\frac{15}{24} \cdot \frac{8}{25}$

60. $-\frac{20}{21} \cdot \frac{7}{16}$

61. $\frac{3}{8} \cdot \frac{7}{16}$

62. $\frac{5}{9} \cdot \frac{2}{7}$

63. $\left(\frac{2}{3}\right)\left(-\frac{1}{16}\right)\left(-\frac{4}{5}\right)$

64. $\left(\frac{3}{8}\right)\left(-\frac{2}{3}\right)\left(-\frac{12}{27}\right)$

65. $-\frac{5}{6} \cdot 18$

66. $6\left(-\frac{2}{3}\right)$

67. $\left(-\frac{3}{4}\right)^3$

68. $\left(-\frac{2}{5}\right)^3$

69. $\frac{3}{4} \cdot \frac{4}{3}$

70. $\frac{4}{5} \cdot \frac{5}{4}$

71. $\frac{5}{3}\left(-\frac{6}{15}\right)(-4)$

72. $\frac{5}{6}\left(-\frac{2}{3}\right)(-12)$

73. $-\frac{11}{12} \cdot \frac{18}{55} \cdot 5$

74. $-\frac{24}{5} \cdot \frac{7}{12} \cdot \frac{1}{14}$

75. $\left(-\frac{11}{21}\right)\left(-\frac{14}{33}\right)$

76. $\left(-\frac{16}{35}\right)\left(-\frac{25}{48}\right)$

77. $-\left(-\frac{5}{9}\right)^2$

78. $-\left(-\frac{5}{6}\right)^2$

79. $\frac{7}{10}\left(\frac{20}{21}\right)$

80. $\left(\frac{7}{6}\right)\frac{9}{49}$

81. $\frac{3}{4}\left(\frac{5}{7}\right)\left(\frac{2}{3}\right)\left(\frac{7}{3}\right)$

82. $-\frac{5}{4}\left(\frac{8}{15}\right)\left(\frac{2}{3}\right)\left(\frac{7}{2}\right)$

83. $-\frac{14}{15}\left(-\frac{11}{8}\right)$

84. $-\frac{5}{16}\left(-\frac{8}{3}\right)$

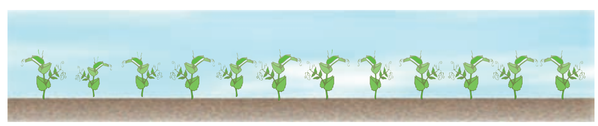
85. $\frac{3}{16} \cdot 4 \cdot \frac{2}{3}$

86. $5 \cdot \frac{7}{5} \cdot \frac{3}{14}$

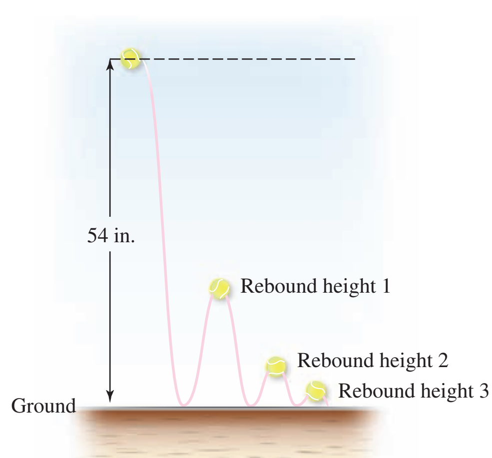
APPLICATIONS

87. SENATE RULES A *filibuster* is a method U.S. Senators sometimes use to block passage of a bill or appointment by talking endlessly. It takes $\frac{3}{5}$ of those voting in the Senate to break a filibuster. If all 100 Senators cast a vote, how many of their votes does it take to break a filibuster?

88. GENETICS Gregor Mendel (1822–1884), an Augustinian monk, is credited with developing a model that became the foundation of modern genetics. In his experiments, he crossed purple-flowered plants with white-flowered plants and found that $\frac{3}{4}$ of the offspring plants had purple flowers and $\frac{1}{4}$ of them had white flowers. Refer to the illustration below, which shows a group of offspring plants. According to this concept, when the plants begin to flower, how many will have purple flowers?



89. BOUNCING BALLS A tennis ball is dropped from a height of 54 inches. Each time it hits the ground, it rebounds one-third of the previous height that it fell. Find the three missing rebound heights in the illustration.



90. ELECTIONS The final election returns for a city bond measure are shown below.

a. Find the total number of votes cast.

b. Find two-thirds of the total number of votes cast.

c. Did the bond measure pass?

MEASURE 1
100% of the precincts reporting

Fire–Police–Paramedics General Obligation Bonds
(Requires two-thirds vote)

YES	NO
125,599	62,801

91. COOKING Use the recipe below, along with the concept of multiplication of fractions, to find how much sugar and how much molasses are needed to make *one dozen* cookies. (*Hint:* this recipe is for *two dozen* cookies.)

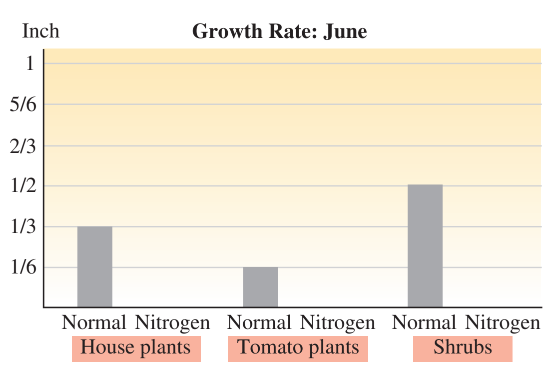
Gingerbread Cookies

$\frac{3}{4}$ cup sugar	$\frac{1}{2}$ cup water
2 cups flour	$\frac{2}{3}$ cup shortening
$\frac{1}{8}$ teaspoon allspice	$\frac{1}{4}$ teaspoon salt
$\frac{1}{3}$ cup dark molasses	$\frac{3}{4}$ teaspoon ginger

Makes two dozen gingerbread cookies.

92. THE EARTH'S SURFACE The surface of Earth covers an area of approximately 196,800,000 square miles. About $\frac{3}{4}$ of that area is covered by water. Find the number of square miles of the surface covered by water.

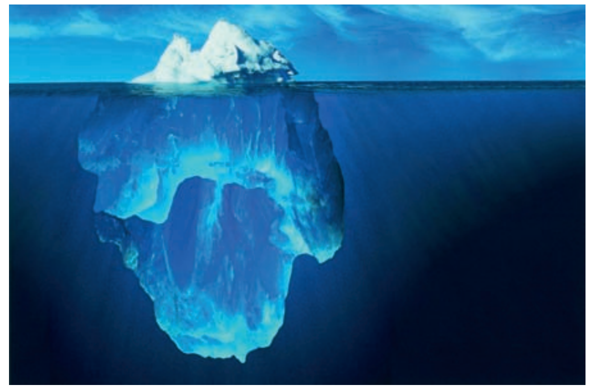
93. BOTANY In an experiment, monthly growth rates of three types of plants doubled when nitrogen was added to the soil. Complete the graph by drawing the improved growth rate bar next to each normal growth rate bar.



94. ICEBERGS About $\frac{9}{10}$ of the volume of an iceberg is below the water line.

a. What fraction of the volume of an iceberg is *above* the water line?

b. Suppose an iceberg has a total volume of 18,700 cubic meters. What is the volume of the part of the iceberg that is above the water line?



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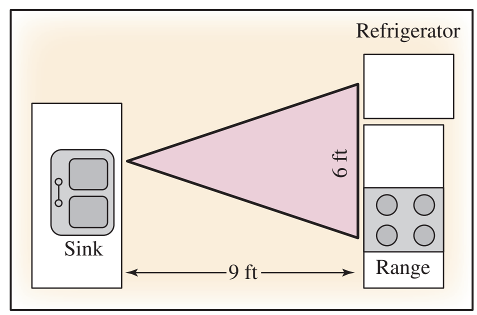
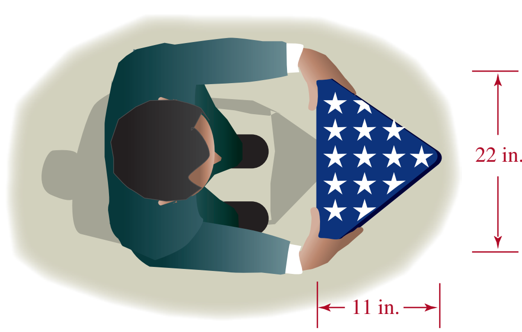
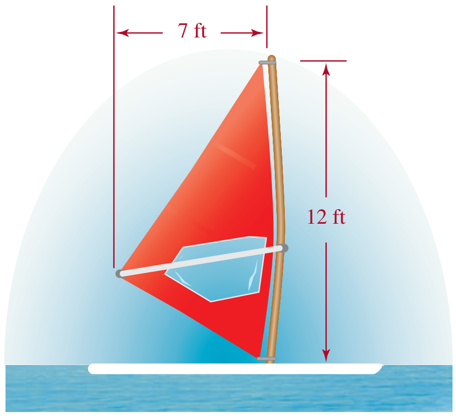
95. KITCHEN DESIGN Find the area of the *kitchen work triangle* formed by the paths between the refrigerator, the range, and the sink shown below.



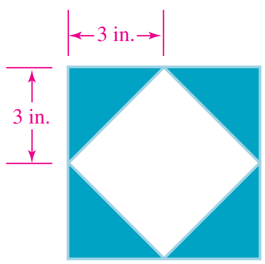
96. STARS AND STRIPES The illustration shows a folded U.S. flag. When it is placed on a table as part of an exhibit, how much area will it occupy?



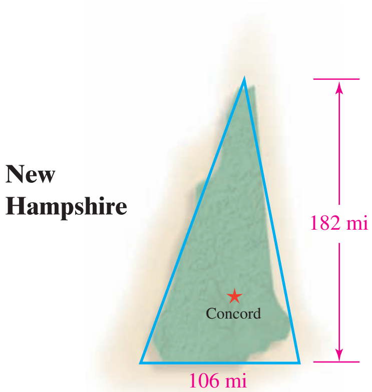
97. WINDSURFING Estimate the area of the sail on the windsurfing board.



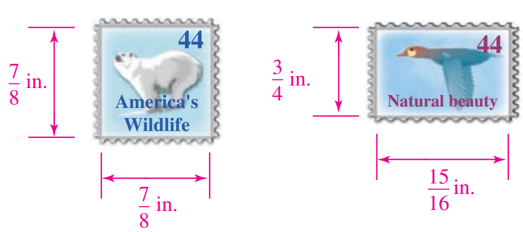
98. TILE DESIGN A design for bathroom tile is shown. Find the amount of area on a tile that is blue.



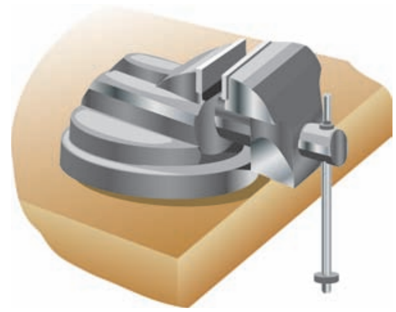
99. GEOGRAPHY Estimate the area of the state of New Hampshire, using the triangle in the illustration.



100. STAMPS The best designs in a contest to create a wildlife stamp are shown. To save on paper costs, the postal service has decided to choose the stamp that has the smaller area. Which one did the postal service choose? (*Hint:* use the formula for the area of a rectangle.)



101. VISES Each complete turn of the handle of the bench vise shown below tightens its jaws exactly $\frac{1}{16}$ of an inch. How much tighter will the jaws of the vice get if the handle is turned 12 complete times?



102. WOODWORKING Each time a board is passed through a power sander, the machine removes $\frac{1}{64}$ of an inch of thickness. If a rough pine board is passed through the sander 6 times, by how much will its thickness change?

WRITING

103. In a word problem, when a fraction is followed by the word *of,* multiplication is usually indicated. Give three real-life examples of this type of use of the word *of.*

104. Can you multiply the number 5 and another number and obtain an answer that is less than 5? Explain why or why not.

105. A MAJORITY The definition of the word *majority* is as follows: "a number greater than *one-half of* the total." Explain what it means when a teacher says, "A majority of the class voted to postpone the test until Monday." Give an example.

106. What does area measure? Give an example.

107. In the following solution, what step did the student forget to use that caused him to have to work with such large numbers?

Multiply. Simplify the product, if possible.

$$\frac{44}{63} \cdot \frac{27}{55} = \frac{44 \cdot 27}{63 \cdot 55}$$
$$= \frac{1{,}188}{3{,}465}$$

108. Is the product of two proper fractions always smaller than either of those fractions? Explain why or why not.

REVIEW

Divide and check each result.

109. $\frac{-8}{4}$

110. $21 \div (-3)$

111. $-736 \div (-32)$

112. $\frac{-400}{-25}$

SECTION 3.3
Dividing Fractions

Objectives

1 Find the reciprocal of a fraction.

2 Divide fractions.

3 Solve application problems by dividing fractions.

We will now discuss how to divide fractions. The fraction multiplication skills that you learned in Section 3.2 will also be useful in this section.

1 Find the reciprocal of a fraction.

Division with fractions involves working with *reciprocals*. To present the concept of reciprocal, we consider the problem $\frac{7}{8} \cdot \frac{8}{7}$.

$$\frac{7}{8} \cdot \frac{8}{7} = \frac{7 \cdot 8}{8 \cdot 7}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{\overset{1}{\cancel{7}} \cdot \overset{1}{\cancel{8}}}{\underset{1}{\cancel{8}} \cdot \underset{1}{\cancel{7}}}$$ To simplify, remove the common factors of 7 and 8 from the numerator and denominator.

$$= \frac{1}{1}$$ Multiply the remaining factors in the numerator. Multiply the remaining factors in the denominator.

$$= 1$$ Any whole number divided by 1 is equal to that number.

The product of $\frac{7}{8}$ and $\frac{8}{7}$ is 1.

Whenever the product of two numbers is 1, we say that those numbers are *reciprocals*. Therefore, $\frac{7}{8}$ and $\frac{8}{7}$ are reciprocals. To find the reciprocal of a fraction, *we invert the numerator and the denominator.*

Reciprocals

Two numbers are called **reciprocals** if their product is 1.

Caution! Zero does not have a reciprocal, because the product of 0 and a number can never be 1.

EXAMPLE 1 For each number, find its reciprocal and show that their product is 1: **a.** $\frac{2}{3}$ **b.** $-\frac{3}{4}$ **c.** 5

Strategy To find each reciprocal, we will invert the numerator and denominator.

WHY This procedure will produce a new fraction that, when multiplied by the original fraction, gives a result of 1.

Solution

a. Fraction **Reciprocal**

$$\frac{2}{3} \xrightarrow{\text{invert}} \frac{3}{2}$$

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

Check: $\frac{2}{3} \cdot \frac{3}{2} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{2}}} = 1$

Self Check 1

For each number, find its reciprocal and show that their product is 1.

a. $\frac{3}{5}$ **b.** $-\frac{5}{6}$ **c.** 8

Now Try **Problem 13**

b. Fraction **Reciprocal**

$$-\frac{3}{4} \xrightarrow{\text{invert}} -\frac{4}{3}$$

The reciprocal of $-\frac{3}{4}$ is $-\frac{4}{3}$.

Check: $-\frac{3}{4}\left(-\frac{4}{3}\right) = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{4}}}{\underset{1}{\cancel{4}} \cdot \underset{1}{\cancel{3}}} = 1$ The product of two fractions with like signs is positive.

c. Since $5 = \frac{5}{1}$, the reciprocal of 5 is $\frac{1}{5}$.

Check: $5 \cdot \frac{1}{5} = \frac{5}{1} \cdot \frac{1}{5} = \frac{\overset{1}{\cancel{5}} \cdot 1}{1 \cdot \underset{1}{\cancel{5}}} = 1$

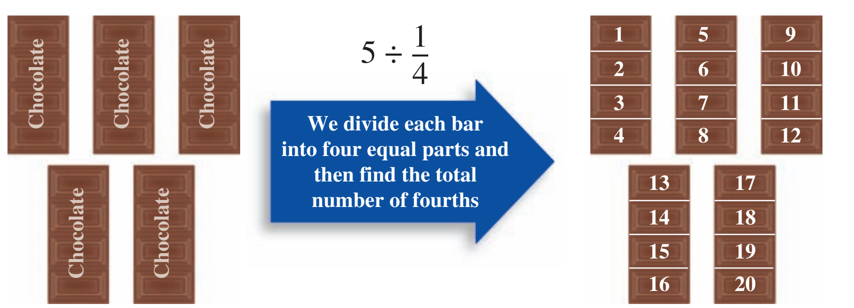
> ***Caution!*** Don't confuse the concepts of the *opposite* of a negative number and the *reciprocal* of a negative number. For example:
>
> The reciprocal of $-\frac{9}{16}$ is $-\frac{16}{9}$.
>
> The opposite of $-\frac{9}{16}$ is $\frac{9}{16}$.

2 Divide fractions.

To develop a rule for dividing fractions, let's consider a real-life application.

Suppose that the manager of a candy store buys large bars of chocolate and divides each one into four equal parts to sell. How many fourths can be obtained from 5 bars?

We are asking, "How many $\frac{1}{4}$'s are there in 5?" To answer the question, we need to use the operation of division. We can represent this division as $5 \div \frac{1}{4}$.



5 bars of chocolate

Total number of fourths = 5 • 4 = 20

There are 20 fourths in the 5 bars of chocolate. Two observations can be made from this result.

- This division problem involves a fraction: $5 \div \frac{1}{4}$.
- Although we were asked to find $5 \div \frac{1}{4}$, we solved the problem using *multiplication* instead of *division:* $5 \cdot 4 = 20$. That is, division by $\frac{1}{4}$ (a fraction) is the same as multiplication by 4 (its reciprocal).

$$5 \div \frac{1}{4} = 5 \cdot 4$$

These observations suggest the following rule for dividing two fractions.

Dividing Fractions

To divide two fractions, multiply the first fraction by the reciprocal of the second fraction. Simplify the result, if possible.

For example, to find $\frac{5}{7} \div \frac{3}{4}$, we multiply $\frac{5}{7}$ by the reciprocal of $\frac{3}{4}$.

Change the division to multiplication.

$$\frac{5}{7} \div \frac{3}{4} = \frac{5}{7} \cdot \frac{4}{3}$$

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

$$= \frac{5 \cdot 4}{7 \cdot 3}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{20}{21}$$

Thus, $\frac{5}{7} \div \frac{3}{4} = \frac{20}{21}$. We say that the *quotient* of $\frac{5}{7}$ and $\frac{3}{4}$ is $\frac{20}{21}$.

EXAMPLE 2 Divide: $\frac{1}{3} \div \frac{4}{5}$

Strategy We will multiply the first fraction, $\frac{1}{3}$, by the reciprocal of the second fraction, $\frac{4}{5}$. Then, if possible, we will simplify the result.

WHY This is the rule for dividing two fractions.

Solution

$$\frac{1}{3} \div \frac{4}{5} = \frac{1}{3} \cdot \frac{5}{4}$$ Multiply $\frac{1}{3}$ by the reciprocal of $\frac{4}{5}$, which is $\frac{5}{4}$.

$$= \frac{1 \cdot 5}{3 \cdot 4}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{5}{12}$$

Since 5 and 12 have no common factors other than 1, the result is in simplest form.

Self Check 2

Divide: $\frac{2}{3} \div \frac{7}{8}$

Now Try **Problem 17**

EXAMPLE 3 Divide and simplify: $\frac{9}{16} \div \frac{3}{20}$

Strategy We will multiply the first fraction, $\frac{9}{16}$, by the reciprocal of the second fraction, $\frac{3}{20}$. Then, if possible, we will simplify the result.

WHY This is the rule for dividing two fractions.

Self Check 3

Divide and simplify: $\frac{4}{5} \div \frac{8}{25}$

Now Try **Problem 21**

Solution

$$\frac{9}{16} \div \frac{3}{20} = \frac{9}{16} \cdot \frac{20}{3}$$ Multiply $\frac{9}{16}$ by the reciprocal of $\frac{3}{20}$, which is $\frac{20}{3}$.

$$= \frac{9 \cdot 20}{16 \cdot 3}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{\overset{1}{\cancel{3}} \cdot 3 \cdot \overset{1}{\cancel{4}} \cdot 5}{\underset{1}{\cancel{4}} \cdot 4 \cdot \underset{1}{\cancel{3}}}$$ To simplify, factor 9 as 3 · 3, factor 20 as 4 · 5, and factor 16 as 4 · 4. Then remove out the common factors of 3 and 4 from the numerator and denominator.

$$= \frac{15}{4}$$ Multiply the remaining factors in the numerator: 1 · 3 · 1 · 5 = 15. Multiply the remaining factors in the denominator: 1 · 4 · 1 = 4.

Self Check 4

Divide and simplify:

$$80 \div \frac{20}{11}$$

Now Try **Problem 27**

EXAMPLE 4

Divide and simplify: $120 \div \frac{10}{7}$

Strategy We will write 120 as a fraction and then multiply the first fraction by the reciprocal of the second fraction.

WHY This is the rule for dividing two fractions.

Solution

$$120 \div \frac{10}{7} = \frac{120}{1} \div \frac{10}{7}$$ Write 120 as a fraction: $120 = \frac{120}{1}$.

$$= \frac{120}{1} \cdot \frac{7}{10}$$ Multiply $\frac{120}{1}$ by the reciprocal of $\frac{10}{7}$, which is $\frac{7}{10}$.

$$= \frac{120 \cdot 7}{1 \cdot 10}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{\overset{1}{\cancel{10}} \cdot 12 \cdot 7}{1 \cdot \underset{1}{\cancel{10}}}$$ To simplify, factor 120 as 10 · 12, then remove the common factor of 10 from the numerator and denominator.

$$= \frac{84}{1}$$ Multiply the remaining factors in the numerator: 1 · 12 · 7 = 84. Multiply the remaining factors in the denominator: 1 · 1 = 1.

$$= 84$$ Any whole number divided by 1 is the same number.

Because of the relationship between multiplication and division, the sign rules for *dividing* fractions are the same as those for *multiplying* fractions.

Self Check 5

Divide and simplify:

$$\frac{2}{3} \div \left(-\frac{7}{6}\right)$$

Now Try **Problem 29**

EXAMPLE 5

Divide and simplify: $\frac{1}{6} \div \left(-\frac{1}{18}\right)$

Strategy We will multiply the first fraction, $\frac{1}{6}$, by the reciprocal of the second fraction, $-\frac{1}{18}$. To determine the sign of the result, we will use the rule for multiplying two fractions that have different (unlike) signs.

WHY One fraction is positive and one is negative.

Solution

$$\frac{1}{6} \div \left(-\frac{1}{18}\right) = \frac{1}{6}\left(-\frac{18}{1}\right)$$
Multiply $\frac{1}{6}$ by the reciprocal of $-\frac{1}{18}$, which is $-\frac{18}{1}$.

$$= -\frac{1 \cdot 18}{6 \cdot 1}$$
Multiply the numerators.
Multiply the denominators.
Since the fractions have unlike signs, make the answer negative.

$$= -\frac{1 \cdot 3 \cdot \overset{1}{\cancel{6}}}{\underset{1}{\cancel{6}} \cdot 1}$$
To simplify, factor 18 as $3 \cdot 6$. Then remove the common factor of 6 from the numerator and denominator.

$$= -\frac{3}{1}$$
Multiply the remaining factors in the numerator.
Multiply the remaining factors in the denominator.

$$= -3$$

EXAMPLE 6 Divide and simplify: $-\frac{21}{36} \div (-3)$

Strategy We will multiply the first fraction, $-\frac{21}{36}$, by the reciprocal of -3. To determine the sign of the result, we will use the rule for multiplying two fractions that have the same (like) signs.

WHY Both fractions are negative.

Solution

$$-\frac{21}{36} \div (-3) = -\frac{21}{36}\left(-\frac{1}{3}\right)$$
Multiply $-\frac{21}{36}$ by the reciprocal of -3, which is $-\frac{1}{3}$.

$$= \frac{21}{36}\left(\frac{1}{3}\right)$$
Since the product of two negative fractions is positive, drop both $-$ signs and continue.

$$= \frac{21 \cdot 1}{36 \cdot 3}$$
Multiply the numerators.
Multiply the denominators.

$$= \frac{\overset{1}{\cancel{3}} \cdot 7 \cdot 1}{36 \cdot \underset{1}{\cancel{3}}}$$
To simplify, factor 21 as $3 \cdot 7$. Then remove the common factor of 3 from the numerator and denominator.

$$= \frac{7}{36}$$
Multiply the remaining factors in the numerator: $1 \cdot 7 \cdot 1 = 7$.
Multiply the remaining factors in the denominator: $36 \cdot 1 = 36$.

Self Check 6

Divide and simplify:

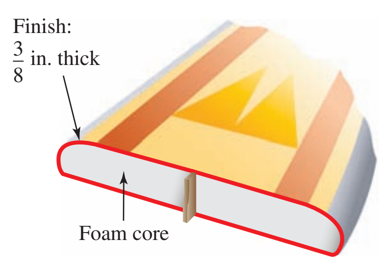
$$-\frac{35}{16} \div (-7)$$

Now Try **Problem 33**

3 Solve application problems by dividing fractions.

Problems that involve forming equal-sized groups can be solved by division.

EXAMPLE 7 ***Surfboard Designs*** Most surfboards are made of a foam core covered with several layers of fiberglass to keep them water-tight. How many layers are needed to build up a finish $\frac{3}{8}$ of an inch thick if each layer of fiberglass has a thickness of $\frac{1}{16}$ of an inch?



Self Check 7

COOKING A recipe calls for 4 cups of sugar, and the only measuring container you have holds $\frac{1}{3}$ cup. How many $\frac{1}{3}$ cups of sugar would you need to add to follow the recipe?

Now Try **Problem 77**

Analyze

- The surfboard is to have a $\frac{3}{8}$-inch-thick fiberglass finish. *Given*
- Each layer of fiberglass is $\frac{1}{16}$ of an inch thick. *Given*
- How many layers of fiberglass need to be applied? *Find*

Form Think of the $\frac{3}{8}$-inch-thick finish separated into an unknown number of equally thick layers of fiberglass. This indicates division.

We translate the words of the problem to numbers and symbols.

The number of layers of fiberglass that are needed	is equal to	the thickness of the finish	divided by	the thickness of 1 layer of fiberglass.
The number of layers of fiberglass that are needed	$=$	$\frac{3}{8}$	$\div$	$\frac{1}{16}$

Solve To find the quotient, we will use the rule for dividing two fractions.

$$\frac{3}{8} \div \frac{1}{16} = \frac{3}{8} \cdot \frac{16}{1}$$ Multiply $\frac{3}{8}$ by the reciprocal of $\frac{1}{16}$, which is $\frac{16}{1}$.

$$= \frac{3 \cdot 16}{8 \cdot 1}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{3 \cdot 2 \cdot \overset{1}{\cancel{8}}}{\underset{1}{\cancel{8}} \cdot 1}$$ To simplify, factor 16 as $2 \cdot 8$. Then remove the common factor of 8 from the numerator and denominator.

$$= \frac{6}{1}$$ Multiply the remaining factors in the numerator. Multiply the remaining factors in the denominator.

$$= 6$$ Any whole number divided by 1 is the same number.

State The number of layers of fiberglass needed is 6.

Check If 6 layers of fiberglass, each $\frac{1}{16}$ of an inch thick, are used, the finished thickness will be $\frac{6}{16}$ of an inch. If we simplify $\frac{6}{16}$, we see that it is equivalent to the desired finish thickness:

$$\frac{6}{16} = \frac{\overset{1}{\cancel{2}} \cdot 3}{\underset{1}{\cancel{2}} \cdot 8} = \frac{3}{8}$$

The result checks.

ANSWERS TO SELF CHECKS

1. a. $\frac{5}{3}$ **b.** $-\frac{6}{5}$ **c.** $\frac{1}{8}$ **2.** $\frac{16}{21}$ **3.** $\frac{5}{2}$ **4.** 44 **5.** $-\frac{4}{7}$ **6.** $\frac{5}{16}$ **7.** 12

SECTION 3.3 STUDY SET

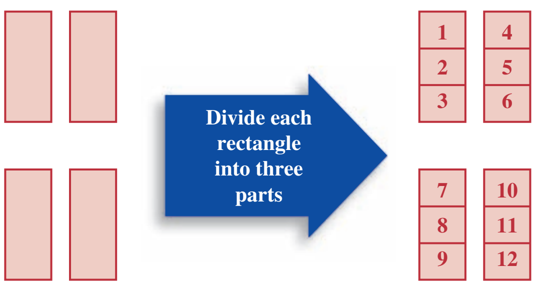
VOCABULARY

Fill in the blanks.

1. The ________ of $\frac{5}{12}$ is $\frac{12}{5}$.
2. To find the reciprocal of a fraction, _______ the numerator and denominator.
3. The answer to a division is called the _________.
4. To simplify $\frac{2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 5 \cdot 7}$, we _______ common factors of the numerator and denominator.

CONCEPTS

5. Fill in the blanks.
   a. To divide two fractions, _________ the first fraction by the _________ of the second fraction.
   b. $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \square \frac{\square}{\square}$
6. a. What division problem is illustrated below?
   b. What is the answer?



7. Determine whether each quotient is positive or negative. *You do not have to find the answer.*
   a. $-\frac{1}{4} \div \frac{3}{4}$
   b. $-\frac{7}{8} \div \left(-\frac{21}{32}\right)$
8. Complete the table.

Number	Opposite	Reciprocal
$\frac{3}{10}$		
$-\frac{7}{11}$		
6		

9. a. Multiply $\frac{4}{5}$ and its reciprocal. What is the result?
   b. Multiply $-\frac{3}{5}$ and its reciprocal. What is the result?
10. a. Find: $15 \div 3$
    b. Rewrite $15 \div 3$ as multiplication by the reciprocal of 3, and find the result.
    c. Complete this statement: Division by 3 is the same as multiplication by $\square$.

NOTATION

Fill in the blanks to complete each solution.

11. $$\begin{aligned} \frac{4}{9} \div \frac{8}{27} &= \frac{4}{9} \cdot \frac{\square}{8} \\ &= \frac{4 \cdot \square}{9 \cdot \square} \\ &= \frac{4 \cdot 3 \cdot \square}{9 \cdot \square \cdot \square} \\ &= \frac{\overset{1}{\cancel{\square}} \cdot 3 \cdot \overset{1}{\cancel{9}}}{\underset{1}{\cancel{\square}} \cdot 2 \cdot \underset{1}{\cancel{4}}} \\ &= \frac{\square}{2} \end{aligned}$$

12. $$\begin{aligned} \frac{25}{31} \div 10 &= \frac{25}{31} \div \frac{10}{\square} \\ &= \frac{25}{31} \cdot \frac{1}{\square} \\ &= \frac{25 \cdot \square}{31 \cdot \square} \\ &= \frac{5 \cdot \square \cdot 1}{31 \cdot 2 \cdot 5} \\ &= \frac{\overset{1}{\cancel{5}} \cdot 5 \cdot 1}{31 \cdot 2 \cdot \underset{1}{\cancel{\square}}} \\ &= \frac{5}{\square} \end{aligned}$$

GUIDED PRACTICE

Find the reciprocal of each number. See Example 1.

13. a. $\frac{6}{7}$ b. $-\frac{15}{8}$ c. 10
14. a. $\frac{2}{9}$ b. $-\frac{9}{4}$ c. 7
15. a. $\frac{11}{8}$ b. $-\frac{1}{14}$ c. -63
16. a. $\frac{13}{2}$ b. $-\frac{1}{5}$ c. -21

Divide. Simplify each quotient, if possible. See Example 2.

17. $\frac{1}{8} \div \frac{2}{3}$
18. $\frac{1}{2} \div \frac{8}{9}$
19. $\frac{2}{23} \div \frac{1}{7}$
20. $\frac{4}{21} \div \frac{1}{5}$

Divide. Simplify each quotient, if possible. **See Example 3.**

21. $\frac{25}{32} \div \frac{5}{28}$

22. $\frac{4}{25} \div \frac{2}{35}$

23. $\frac{27}{32} \div \frac{9}{8}$

24. $\frac{16}{27} \div \frac{20}{21}$

Divide. Simplify each quotient, if possible. **See Example 4.**

25. $50 \div \frac{10}{9}$

26. $60 \div \frac{10}{3}$

27. $150 \div \frac{15}{32}$

28. $170 \div \frac{17}{6}$

Divide. Simplify each quotient, if possible. **See Example 5.**

29. $\frac{1}{8} \div \left(-\frac{1}{32}\right)$

30. $\frac{1}{9} \div \left(-\frac{1}{27}\right)$

31. $\frac{2}{5} \div \left(-\frac{4}{35}\right)$

32. $\frac{4}{9} \div \left(-\frac{16}{27}\right)$

Divide. Simplify each quotient, if possible. **See Example 6.**

33. $-\frac{28}{55} \div (-7)$

34. $-\frac{32}{45} \div (-8)$

35. $-\frac{33}{23} \div (-11)$

36. $-\frac{21}{31} \div (-7)$

TRY IT YOURSELF

Divide. Simplify each quotient, if possible.

37. $120 \div \frac{12}{5}$

38. $360 \div \frac{36}{5}$

39. $\frac{1}{2} \div \frac{3}{5}$

40. $\frac{1}{7} \div \frac{5}{6}$

41. $\left(-\frac{7}{4}\right) \div \left(-\frac{21}{8}\right)$

42. $\left(-\frac{15}{16}\right) \div \left(-\frac{5}{8}\right)$

43. $\frac{4}{5} \div \frac{4}{5}$

44. $\frac{2}{3} \div \frac{2}{3}$

45. Divide $-\frac{15}{32}$ by $\frac{3}{4}$

46. Divide $-\frac{7}{10}$ by $\frac{4}{5}$

47. $3 \div \frac{1}{12}$

48. $9 \div \frac{3}{4}$

49. $-\frac{4}{5} \div (-6)$

50. $-\frac{7}{8} \div (-14)$

51. $\frac{15}{16} \div 180$

52. $\frac{7}{8} \div 210$

53. $-\frac{9}{10} \div \frac{4}{15}$

54. $-\frac{3}{4} \div \frac{3}{2}$

55. $\frac{9}{10} \div \left(-\frac{3}{25}\right)$

56. $\frac{11}{16} \div \left(-\frac{9}{16}\right)$

57. $\frac{3}{16} \div \frac{1}{9}$

58. $\frac{5}{8} \div \frac{2}{9}$

59. $-\frac{1}{8} \div 8$

60. $-\frac{1}{15} \div 15$

The following problems involve multiplication and division. Perform each operation. Simplify the result, if possible.

61. $\frac{7}{6} \cdot \frac{9}{49}$

62. $\frac{7}{10} \cdot \frac{20}{21}$

63. $-\frac{4}{5} \div \left(-\frac{3}{2}\right)$

64. $-\frac{2}{3} \div \left(-\frac{3}{2}\right)$

65. $\frac{13}{16} \div 2$

66. $\frac{7}{8} \div 6$

67. $\left(-\frac{11}{21}\right)\left(-\frac{14}{33}\right)$

68. $\left(-\frac{16}{35}\right)\left(-\frac{25}{48}\right)$

69. $-\frac{15}{32} \div \frac{5}{64}$

70. $-\frac{28}{15} \div \frac{21}{10}$

71. $11 \cdot \frac{1}{6}$

72. $9 \cdot \frac{1}{8}$

73. $\frac{3}{4} \cdot \frac{5}{7}$

74. $\frac{2}{3} \cdot \frac{7}{9}$

75. $\frac{25}{7} \div \left(-\frac{30}{21}\right)$

76. $\frac{39}{25} \div \left(-\frac{13}{10}\right)$

APPLICATIONS

77. PATIO FURNITURE A production process applies several layers of a clear plastic coat to outdoor furniture to help protect it from the weather. If each protective coat is $\frac{3}{32}$-inch thick, how many applications will be needed to build up $\frac{3}{8}$ inch of clear finish?

78. MARATHONS Each lap around a stadium track is $\frac{1}{4}$ mile. How many laps would a runner have to complete to get a 26-mile workout?

79. COOKING A recipe calls for $\frac{3}{4}$ cup of flour, and the only measuring container you have holds $\frac{1}{8}$ cup. How many $\frac{1}{8}$ cups of flour would you need to add to follow the recipe?

80. LASERS A technician uses a laser to slice thin pieces of aluminum off the end of a rod that is $\frac{7}{8}$-inch long. How many $\frac{1}{64}$-inch-wide slices can be cut from this rod? (Assume that there is no waste in the process.)

81. UNDERGROUND CABLES Refer to the illustration and table on the next page.

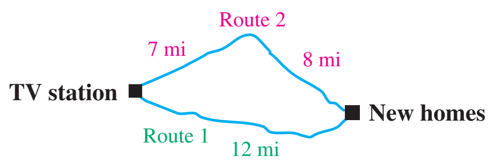
a. How many days will it take to install underground TV cable from the broadcasting station to the new homes using route 1?

b. How long is route 2?

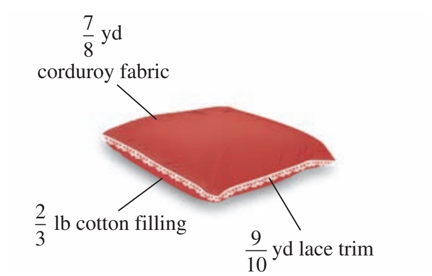
c. How many days will it take to install the cable using route 2?

d. Which route will require the fewer number of days to install the cable?

Proposal	Amount of cable installed per day	Comments
Route 1	$\frac{2}{5}$ of a mile	Ground very rocky
Route 2	$\frac{3}{5}$ of a mile	Longer than Route 1



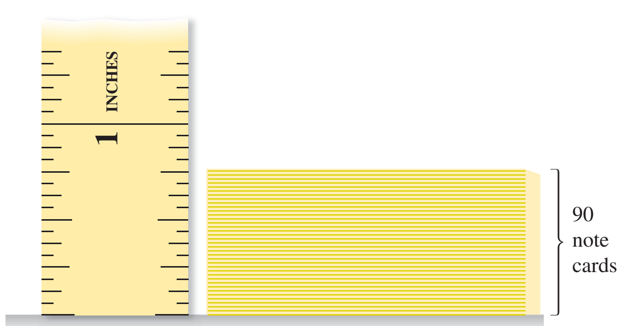
82. PRODUCTION PLANNING The materials used to make a pillow are shown. Examine the inventory list to decide how many pillows can be manufactured in one production run with the materials in stock.



Factory Inventory List

Materials	Amount in stock
Lace trim	135 yd
Corduroy fabric	154 yd
Cotton filling	98 lb

83. NOTE CARDS Ninety 3 × 5 cards are stacked next to a ruler as shown.

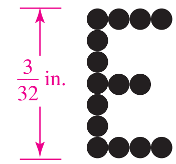


a. Into how many parts is 1 inch divided on the ruler?

b. How thick is the stack of cards?

c. How thick is one 3 × 5 card?

84. COMPUTER PRINTERS The illustration shows how the letter E is formed by a dot matrix printer. What is the height of one dot?



85. FORESTRY A set of forestry maps divides the 6,284 acres of an old-growth forest into $\frac{4}{5}$-acre sections. How many sections do the maps contain?

86. HARDWARE A hardware chain purchases large amounts of nails and packages them in $\frac{9}{16}$-pound bags for sale. How many of these bags of nails can be obtained from 2,871 pounds of nails?

WRITING

87. Explain how to divide two fractions.

88. Why do you need to know how to multiply fractions to be able to divide fractions?

89. Explain why 0 does not have a reciprocal.

90. What number is its own reciprocal? Explain why this is so.

91. Write an application problem that could be solved by finding $10 \div \frac{1}{5}$.

92. Explain why dividing a fraction by 2 is the same as finding $\frac{1}{2}$ of it. Give an example.

REVIEW

Fill in the blanks.

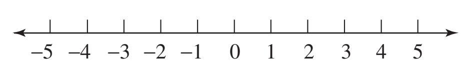
93. The symbol $<$ means ___ _____ _____.

94. The statement $9 \cdot 8 = 8 \cdot 9$ illustrates the ___________ property of multiplication.

95. ______ is neither positive nor negative.

96. The sum of two negative numbers is ________.

97. Graph each of these numbers on a number line: -2, 0, $|-4|$, and the opposite of 1



98. Evaluate each expression.

a. 3^5 **b.** $(-2)^5$

Objectives

1. Add and subtract fractions that have the same denominator.
2. Add and subtract fractions that have different denominators.
3. Find the LCD to add and subtract fractions.
4. Identify the greater of two fractions.
5. Solve application problems by adding and subtracting fractions.

SECTION 3.4
Adding and Subtracting Fractions

In mathematics and everyday life, we can only add (or subtract) objects that are similar. For example, we can add dollars to dollars, but we cannot add dollars to oranges. This concept is important when adding or subtracting fractions.

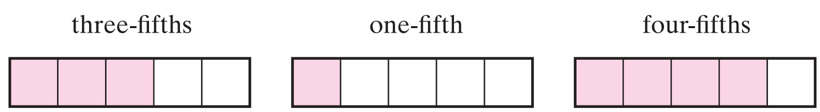
1 Add and subtract fractions that have the same denominator.

Consider the problem $\frac{3}{5} + \frac{1}{5}$. When we write it in words, it is apparent that we are adding similar objects.

three-**fifths** + one-**fifth**

Similar objects

Because the denominators of $\frac{3}{5}$ and $\frac{1}{5}$ are the same, we say that they have a **common denominator.** Since the fractions have a common denominator, we can add them. The following figure explains the addition process.



$$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

We can make some observations about the addition shown in the figure.

The sum of the numerators is the numerator of the answer.

$$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

The answer is a fraction that has the same denominator as the two fractions that were added.

These observations illustrate the following rule.

Adding and Subtracting Fractions That Have the Same Denominator

To add (or subtract) fractions that have the same denominator, add (or subtract) their numerators and write the sum (or difference) over the common denominator. Simplify the result, if possible.

Caution! We **do not** add fractions by adding the numerators and adding the denominators!

$$\cancel{\frac{3}{5} + \frac{1}{5} = \frac{3+1}{5+5} = \frac{4}{10}}$$

The same caution applies when subtracting fractions.

EXAMPLE 1

Perform each operation and simplify the result, if possible.

a. Add: $\frac{1}{8} + \frac{5}{8}$ **b.** Subtract: $\frac{11}{15} - \frac{4}{15}$

Strategy We will use the rule for adding and subtracting fractions that have *the same* denominator.

WHY In part a, the fractions have the same denominator, 8. In part b, the fractions have the same denominator, 15.

Solution

a.
$$\frac{1}{8} + \frac{5}{8} = \frac{1+5}{8}$$
Add the numerators and write the sum over the common denominator 8.

$$= \frac{6}{8}$$
This fraction can be simplified.

$$= \frac{\overset{1}{\cancel{2}} \cdot 3}{\underset{1}{\cancel{2}} \cdot 4}$$
To simplify, factor 6 as $2 \cdot 3$ and 8 as $2 \cdot 4$. Then remove the common factor of 2 from the numerator and denominator.

$$= \frac{3}{4}$$
Multiply the remaining factors in the numerator: $1 \cdot 3 = 3$.
Multiply the remaining factors in the denominator: $1 \cdot 4 = 4$.

b.
$$\frac{11}{15} - \frac{4}{15} = \frac{11-4}{15}$$
Subtract the numerators and write the difference over the common denominator 15.

$$= \frac{7}{15}$$

Since 7 and 15 have no common factors other than 1, the result is in simplest form.

The rule for subtraction from Section 2.3 can be extended to subtraction involving signed fractions:

To subtract two fractions, add the first to the opposite of the fraction to be subtracted.

EXAMPLE 2

Subtract: $-\frac{7}{3} - \left(-\frac{2}{3}\right)$

Strategy To find the difference, we will apply the rule for subtraction.

WHY It is easy to make an error when subtracting signed fractions. We will probably be more accurate if we write the subtraction as addition of the opposite.

Solution

We read $-\frac{7}{3} - \left(-\frac{2}{3}\right)$ as "negative seven-thirds *minus* negative two-thirds." Thus, the number to be subtracted is $-\frac{2}{3}$. Subtracting $-\frac{2}{3}$ is the same as adding its opposite, $\frac{2}{3}$.

Add

$$-\frac{7}{3} - \left(-\frac{2}{3}\right) = -\frac{7}{3} + \frac{2}{3}$$
Add the opposite of $-\frac{2}{3}$, which is $\frac{2}{3}$.

the opposite

$$= \frac{-7}{3} + \frac{2}{3}$$
Write $-\frac{7}{3}$ as $\frac{-7}{3}$.

$$= \frac{-7+2}{3}$$
Add the numerators and write the sum over the common denominator 3.

$$= \frac{-5}{3}$$
Use the rule for adding two integers with different signs: $-7 + 2 = -5$.

$$= -\frac{5}{3}$$
Rewrite the result with the $-$ sign in front: $\frac{-5}{3} = -\frac{5}{3}$. This fraction is in simplest form.

Self Check 1

Perform each operation and simplify the result, if possible.

a. Add: $\frac{5}{12} + \frac{1}{12}$

b. Subtract: $\frac{8}{9} - \frac{1}{9}$

Now Try **Problems 17 and 21**

Self Check 2

Subtract: $-\frac{9}{11} - \left(-\frac{3}{11}\right)$

Now Try **Problem 25**

Self Check 3

Perform the operations and simplify:

$$\frac{2}{9}+\frac{2}{9}+\frac{2}{9}$$

Now Try **Problem 29**

EXAMPLE 3 Perform the operations and simplify: $\frac{18}{25}-\frac{2}{25}-\frac{1}{25}$

Strategy We will use the rule for subtracting fractions that have *the same* denominator.

WHY All three fractions have the same denominator, 25.

Solution

$$\frac{18}{25}-\frac{2}{25}-\frac{1}{25}=\frac{18-2-1}{25}$$ Subtract the numerators and write the difference over the common denominator 25.

$$=\frac{15}{25}$$ This fraction can be simplified.

$$=\frac{3\cdot\overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}}\cdot 5}$$ To simplify, factor 15 as 3 · 5 and 25 as 5 · 5. Then remove the common factor of 5 from the numerator and denominator.

$$=\frac{3}{5}$$ Multiply the remaining factors in the numerator: 3 · 1 = 3. Multiply the remaining factors in the denominator: 1 · 5 = 5.

2 Add and subtract fractions that have different denominators.

Now we consider the problem $\frac{3}{5}+\frac{1}{3}$. Since the denominators are different, we cannot add these fractions in their present form.

three-**fifths** + one-**third**

Not similar objects

To add (or subtract) fractions with different denominators, we express them as equivalent fractions that have a common denominator. The smallest common denominator, called the **least** or **lowest common denominator,** is usually the easiest common denominator to use.

Least Common Denominator

The **least common denominator (LCD)** for a set of fractions is the smallest number each denominator will divide exactly (divide with no remainder).

The denominators of $\frac{3}{5}$ and $\frac{1}{3}$ are 5 and 3. The numbers 5 and 3 divide many numbers exactly (30, 45, and 60, to name a few), but the smallest number that they divide exactly is 15. Thus, 15 is the LCD for $\frac{3}{5}$ and $\frac{1}{3}$.

To find $\frac{3}{5}+\frac{1}{3}$, we *build* equivalent fractions that have denominators of 15. (This procedure was introduced in Section 3.1.) Then we use the rule for adding fractions that have the same denominator.

$$\frac{3}{5}+\frac{1}{3}=\frac{3}{5}\cdot\frac{3}{3}+\frac{1}{3}\cdot\frac{5}{5}$$

We need to multiply this denominator by 5 to obtain 15. It follows that $\frac{5}{5}$ should be the form of 1 used to build $\frac{1}{3}$.

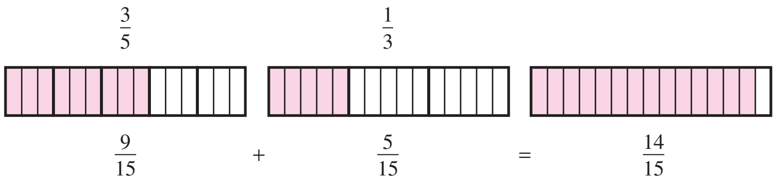
We need to multiply this denominator by 3 to obtain 15. It follows that $\frac{3}{3}$ should be the form of 1 that is used to build $\frac{3}{5}$.

$$=\frac{9}{15}+\frac{5}{15}$$ Multiply the numerators. Multiply the denominators. Note that the denominators are now the same.

$$=\frac{9+5}{15}$$ Add the numerators and write the sum over the common denominator 15.

$$=\frac{14}{15}$$ Since 14 and 15 have no common factors other than 1, this fraction is in simplest form.

The figure below shows $\frac{3}{5}$ and $\frac{1}{3}$ expressed as equivalent fractions with a denominator of 15. Once the denominators are the same, the fractions are similar objects and can be added easily.



We can use the following steps to add or subtract fractions with different denominators.

Adding and Subtracting Fractions That Have Different Denominators

1. Find the LCD.
2. Rewrite each fraction as an equivalent fraction with the LCD as the denominator. To do so, build each fraction using a form of 1 that involves any factors needed to obtain the LCD.
3. Add or subtract the numerators and write the sum or difference over the LCD.
4. Simplify the result, if possible.

EXAMPLE 4

Add: $\frac{1}{7} + \frac{2}{3}$

Strategy We will express each fraction as an equivalent fraction that has the LCD as its denominator. Then we will use the rule for adding fractions that have the same denominator.

WHY To add (or subtract) fractions, the fractions must have *like* denominators.

Solution

Since the smallest number the denominators 7 and 3 divide exactly is 21, the LCD is 21.

$$\frac{1}{7} + \frac{2}{3} = \frac{1}{7} \cdot \frac{3}{3} + \frac{2}{3} \cdot \frac{7}{7}$$
To build $\frac{1}{7}$ and $\frac{2}{3}$ so that their denominators are 21, multiply each by a form of 1.

$$= \frac{3}{21} + \frac{14}{21}$$
Multiply the numerators. Multiply the denominators. The denominators are now the same.

$$= \frac{3 + 14}{21}$$
Add the numerators and write the sum over the common denominator 21.

$$= \frac{17}{21}$$
Since 17 and 21 have no common factors other than 1, this fraction is in simplest form.

Self Check 4

Add: $\frac{1}{2} + \frac{2}{5}$

Now Try **Problem 35**

EXAMPLE 5

Subtract: $\frac{5}{2} - \frac{7}{3}$

Strategy We will express each fraction as an equivalent fraction that has the LCD as its denominator. Then we will use the rule for subtracting fractions that have the same denominator.

Self Check 5

Subtract: $\frac{6}{7} - \frac{3}{5}$

Now Try **Problem 37**

WHY To add (or subtract) fractions, the fractions must have *like* denominators.

Solution

Since the smallest number the denominators 2 and 3 divide exactly is 6, the LCD is 6.

$$\frac{5}{2} - \frac{7}{3} = \frac{5}{2} \cdot \frac{3}{3} - \frac{7}{3} \cdot \frac{2}{2}$$ To build $\frac{5}{2}$ and $\frac{7}{3}$ so that their denominators are 6, multiply each by a form of 1.

$$= \frac{15}{6} - \frac{14}{6}$$ Multiply the numerators. Multiply the denominators. The denominators are now the same.

$$= \frac{15 - 14}{6}$$ Subtract the numerators and write the difference over the common denominator 6.

$$= \frac{1}{6}$$ This fraction is in simplest form.

Self Check 6

Subtract: $\frac{2}{3} - \frac{13}{6}$

Now Try **Problem 41**

EXAMPLE 6 Subtract: $\frac{2}{5} - \frac{11}{15}$

Strategy Since the smallest number the denominators 5 and 15 divide exactly is 15, the LCD is 15. We will only need to build an equivalent fraction for $\frac{2}{5}$.

WHY We do not have to build the fraction $\frac{11}{15}$ because it already has a denominator of 15.

Solution

$$\frac{2}{5} - \frac{11}{15} = \frac{2}{5} \cdot \frac{3}{3} - \frac{11}{15}$$ To build $\frac{2}{5}$ so that its denominator is 15, multiply it by a form of 1.

$$= \frac{6}{15} - \frac{11}{15}$$ Multiply the numerators. Multiply the denominators. The denominators are now the same.

$$= \frac{6 - 11}{15}$$ Subtract the numerators and write the difference over the common denominator 15.

$$= -\frac{5}{15}$$ If it is helpful, use the subtraction rule and add the opposite in the numerator: $6 + (-11) = -5$. Write the $-$ sign in front of the fraction.

$$= -\frac{\overset{1}{\cancel{5}}}{3 \cdot \underset{1}{\cancel{5}}}$$ To simplify, factor 15 as $3 \cdot 5$. Then remove the common factor of 5 from the numerator and denominator.

$$= -\frac{1}{3}$$ Multiply the remaining factors in the denominator: $3 \cdot 1 = 3$.

Success Tip In Example 6, did you notice that the denominator 5 is a factor of the denominator 15, and that the LCD is 15. In general, when adding (or subtracting) two fractions with different denominators, *if the smaller denominator is a factor of the larger denominator, the larger denominator is the LCD.*

Caution! You might not have to build each fraction when adding or subtracting fractions with different denominators. For instance, the step in blue shown below is unnecessary when solving Example 6.

$$\frac{2}{5} - \frac{11}{15} = \frac{2}{5} \cdot \frac{3}{3} - \frac{11}{15} \cdot \cancel{\frac{1}{1}}$$

EXAMPLE 7 Add: $-5 + \frac{3}{4}$

Strategy We will write -5 as the fraction $\frac{-5}{1}$. Then we will follow the steps for adding fractions that have different denominators.

WHY The fractions $\frac{-5}{1}$ and $\frac{3}{4}$ have different denominators.

Solution

Since the smallest number the denominators 1 and 4 divide exactly is 4, the LCD is 4.

$$-5 + \frac{3}{4} = \frac{-5}{1} + \frac{3}{4}$$ Write -5 as $\frac{-5}{1}$.

$$= \frac{-5}{1} \cdot \frac{4}{4} + \frac{3}{4}$$ To build $\frac{-5}{1}$ so that its denominator is 4, multiply it by a form of 1.

$$= \frac{-20}{4} + \frac{3}{4}$$ Multiply the numerators. Multiply the denominators. The denominators are now the same.

$$= \frac{-20 + 3}{4}$$ Add the numerators and write the sum over the common denominator 4.

$$= \frac{-17}{4}$$ Use the rule for adding two integers with different signs: $-20 + 3 = -17$.

$$= -\frac{17}{4}$$ Write the result with the $-$ sign in front: $\frac{-17}{4} = -\frac{17}{4}$. This fraction is in simplest form.

Self Check 7

Add: $-6 + \frac{3}{8}$

Now Try **Problem 45**

3 Find the LCD to add and subtract fractions.

When we add or subtract fractions that have different denominators, the least common denominator is not always obvious. We can use a concept studied earlier to determine the LCD for more difficult problems that involve larger denominators. To illustrate this, let's find the least common denominator of $\frac{3}{8}$ and $\frac{1}{10}$. (Note, the LCD *is not* 80.)

We have learned that both 8 and 10 must divide the LCD exactly. This divisibility requirement should sound familiar. Recall the following fact from Section 1.8.

The Least Common Multiple (LCM)

The **least common multiple (LCM)** of two whole numbers is the smallest whole number that is divisible by both of those numbers.

Thus, the least common denominator of $\frac{3}{8}$ and $\frac{1}{10}$ is simply the *least common multiple* of 8 and 10.

We can find the LCM of 8 and 10 by listing multiples of the larger number, 10, until we find one that is divisible by the smaller number, 8. (This method is explained in Example 2 of Section 1.8.)

Multiples of 10: 10, 20, 30, **40**, 50, 60, . . .

This is the first multiple of 10 that is divisible by 8 (no remainder).

Since the LCM of 8 and 10 is 40, it follows that the LCD of $\frac{3}{8}$ and $\frac{1}{10}$ is 40.

We can also find the LCM of 8 and 10 using prime factorization. We begin by prime factoring 8 and 10. (This method is explained in Example 4 of Section 1.8.)

$$8 = 2 \cdot 2 \cdot 2$$
$$10 = 2 \cdot 5$$

The LCM of 8 and 10 is a product of prime factors, where each factor is used the greatest number of times it appears in any one factorization.

- We will use the factor 2 three times, because 2 appears three times in the factorization of 8. Circle $2 \cdot 2 \cdot 2$, as shown on the previous page.
- We will use the factor 5 once, because it appears one time in the factorization of 10. Circle 5 as shown on the previous page.

Since there are no other prime factors in either prime factorization, we have

$$\text{LCM}(8, 10) = \underbrace{2 \cdot 2 \cdot 2}_{\text{Use 2 three times.}} \cdot \underbrace{5}_{\text{Use 5 one time.}} = 40$$

Finding the LCD

The least common denominator (LCD) of a set of fractions is the least common multiple (LCM) of the denominators of the fractions. Two ways to find the LCM of the denominators are as follows:

- Write the multiples of the largest denominator in increasing order, until one is found that is divisible by the other denominators.
- Prime factor each denominator. The LCM is a product of prime factors, where each factor is used the greatest number of times it appears in any one factorization.

Self Check 8

Add: $\frac{1}{8} + \frac{5}{6}$

Now Try **Problem 49**

EXAMPLE 8

Add: $\frac{7}{15} + \frac{3}{10}$

Strategy We begin by expressing each fraction as an equivalent fraction that has the LCD for its denominator. Then we use the rule for adding fractions that have the same denominator.

WHY To add (or subtract) fractions, the fractions must have *like* denominators.

Solution

To find the LCD, we find the prime factorization of both denominators and use each prime factor the *greatest* number of times it appears in any one factorization:

$$\left.\begin{aligned} 15 &= 3 \cdot 5 \\ 10 &= 2 \cdot 5 \end{aligned}\right\} \text{LCD} = 2 \cdot 3 \cdot 5 = 30$$

2 appears once in the factorization of 10.
3 appears once in the factorization of 15.
5 appears once in the factorizations of 15 and 10.

The LCD for $\frac{7}{15}$ and $\frac{3}{10}$ is 30.

$$\frac{7}{15} + \frac{3}{10} = \frac{7}{15} \cdot \frac{2}{2} + \frac{3}{10} \cdot \frac{3}{3}$$

To build $\frac{7}{15}$ and $\frac{3}{10}$ so that their denominators are 30, multiply each by a form of 1.

$$= \frac{14}{30} + \frac{9}{30}$$

Multiply the numerators. Multiply the denominators. The denominators are now the same.

$$= \frac{14 + 9}{30}$$

Add the numerators and write the sum over the common denominator 30.

$$= \frac{23}{30}$$

Since 23 and 30 have no common factors other than 1, this fraction is in simplest form.

EXAMPLE 9 Subtract and simplify: $\frac{13}{28} - \frac{1}{21}$

Strategy We begin by expressing each fraction as an equivalent fraction that has the LCD for its denominator. Then we use the rule for subtracting fractions with *like* denominators.

WHY To add (or subtract) fractions, the fractions must have like denominators.

Solution

To find the LCD, we find the prime factorization of both denominators and use each prime factor the *greatest* number of times it appears in any one factorization:

$$\left.\begin{aligned} 28 &= 2 \cdot 2 \cdot 7 \\ 21 &= 3 \cdot 7 \end{aligned}\right\} \text{LCD} = 2 \cdot 2 \cdot 3 \cdot 7 = 84$$

2 appears twice in the factorization of 28.
3 appears once in the factorization of 21.
7 appears once in the factorizations of 28 and 21.

The LCD for $\frac{13}{28}$ and $\frac{1}{21}$ is 84.

We will compare the prime factorizations of 28, 21, and the prime factorization of the LCD, 84, to determine what forms of 1 to use to build equivalent fractions for $\frac{13}{28}$ and $\frac{1}{21}$ with a denominator of 84.

$$\text{LCD} = 2 \cdot 2 \cdot 3 \cdot 7$$

Cover the prime factorization of 28. Since 3 is left uncovered, use $\frac{3}{3}$ to build $\frac{13}{28}$.

$$\text{LCD} = 2 \cdot 2 \cdot 3 \cdot 7$$

Cover the prime factorization of 21. Since $2 \cdot 2 = 4$ is left uncovered, use $\frac{4}{4}$ to build $\frac{1}{21}$.

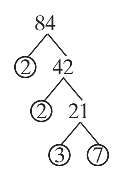
$$\frac{13}{28} - \frac{1}{21} = \frac{13}{28} \cdot \frac{3}{3} - \frac{1}{21} \cdot \frac{4}{4}$$ To build $\frac{13}{28}$ and $\frac{1}{21}$ so that their denominators are 84, multiply each by a form of 1.

$$= \frac{39}{84} - \frac{4}{84}$$ Multiply the numerators. Multiply the denominators. The denominators are now the same.

$$= \frac{39 - 4}{84}$$ Subtract the numerators and write the difference over the common denominator.

$$= \frac{35}{84}$$ This fraction is not in simplest form.

$$= \frac{5 \cdot \overset{1}{\cancel{7}}}{2 \cdot 2 \cdot 3 \cdot \underset{1}{\cancel{7}}}$$ To simplify, factor 35 and 84. Then remove the common factor of 7 from the numerator and denominator.



$$= \frac{5}{12}$$ Multiply the remaining factors in the numerator: $5 \cdot 1 = 5$. Multiply the remaining factors in the denominator: $2 \cdot 2 \cdot 3 \cdot 1 = 12$.

4 Identify the greater of two fractions.

If two fractions have the same denominator, the fraction with the greater numerator is the greater fraction.

For example,

$$\frac{7}{8} > \frac{3}{8} \quad \text{because } 7 > 3 \qquad -\frac{1}{3} > -\frac{2}{3} \quad \text{because } -1 > -2$$

If the denominators of two fractions are different, we need to write the fractions with a common denominator (preferably the LCD) before we can make a comparison.

Self Check 9

Subtract and simplify: $\frac{21}{56} - \frac{9}{40}$

Now Try **Problem 53**

Self Check 10

Which fraction is larger:

$\frac{7}{12}$ or $\frac{3}{5}$?

Now Try **Problem 61**

EXAMPLE 10 Which fraction is larger: $\frac{5}{6}$ or $\frac{7}{8}$?

Strategy We will express each fraction as an equivalent fraction that has the LCD for its denominator. Then we will compare their numerators.

WHY We cannot compare the fractions as given. They are not similar objects.

five-**sixths** seven-**eighths**

Solution

Since the smallest number the denominators will divide exactly is 24, the LCD for $\frac{5}{6}$ and $\frac{7}{8}$ is 24.

$$\frac{5}{6} = \frac{5}{6} \cdot \frac{4}{4} = \frac{20}{24} \qquad \frac{7}{8} = \frac{7}{8} \cdot \frac{3}{3} = \frac{21}{24}$$

To build $\frac{5}{6}$ and $\frac{7}{8}$ so that their denominators are 24, multiply each by a form of 1.

Multiply the numerators.
Multiply the denominators.

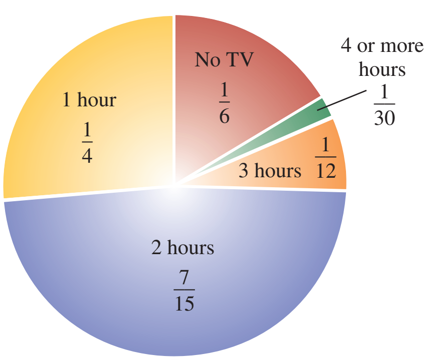
Next, we compare the numerators. Since $21 > 20$, it follows that $\frac{21}{24}$ is greater than $\frac{20}{24}$. Thus, $\frac{7}{8} > \frac{5}{6}$.

5 Solve application problems by adding and subtracting fractions.

Self Check 11

Refer to the circle graph for Example 11. Find the fraction of the student body that watches 2 or more hours of television daily.

Now Try **Problems 65 and 109**



EXAMPLE 11 ***Television Viewing Habits*** Students on a college campus were asked to estimate to the nearest hour how much television they watched each day. The results are given in the **circle graph** below (also called a **pie chart**). For example, the chart tells us that $\frac{1}{4}$ of those responding watched 1 hour per day. What fraction of the student body watches from 0 to 2 hours daily?

Analyze

- $\frac{1}{6}$ of the student body watches no TV daily. *Given*
- $\frac{1}{4}$ of the student body watches 1 hour of TV daily. *Given*
- $\frac{7}{15}$ of the student body watches 2 hours of TV daily. *Given*
- What fraction of the student body watches 0 to 2 hours of TV daily? *Find*

Form We translate the words of the problem to numbers and symbols.

The fraction of the student body that watches from 0 to 2 hours of TV daily	is equal to	the fraction that watches no TV daily	plus	the fraction that watches 1 hour of TV daily	plus	the fraction that watches 2 hours of TV daily.
The fraction of the student body that watches from 0 to 2 hours of TV daily	$=$	$\frac{1}{6}$	$+$	$\frac{1}{4}$	$+$	$\frac{7}{15}$

Solve We must find the sum of three fractions with different denominators. To find the LCD, we prime factor the denominators and use each prime factor the *greatest* number of times it appears in any one factorization:

$$\left.\begin{aligned} 6 &= 2 \cdot 3 \\ 4 &= 2 \cdot 2 \\ 15 &= 3 \cdot 5 \end{aligned}\right\} \text{LCD} = 2 \cdot 2 \cdot 3 \cdot 5 = 60$$

2 appears twice in the factorization of 4.
3 appears once in the factorization of 6 and 15.
5 appears once in the factorization of 15.

The LCD for $\frac{1}{6}$, $\frac{1}{4}$, and $\frac{7}{15}$ is 60.

$$\frac{1}{6} + \frac{1}{4} + \frac{7}{15} = \frac{1}{6} \cdot \frac{10}{10} + \frac{1}{4} \cdot \frac{15}{15} + \frac{7}{15} \cdot \frac{4}{4}$$

Build each fraction so that its denominator is 60.

$$= \frac{10}{60} + \frac{15}{60} + \frac{28}{60}$$

Multiply the numerators. Multiply the denominators. The denominators are now the same.

$$= \frac{10 + 15 + 28}{60}$$

Add the numerators and write the sum over the common denominator 60.

$$\begin{array}{r} \overset{1}{1}0 \\ 15 \\ +\ 28 \\ \hline 53 \end{array}$$

$$= \frac{53}{60}$$

This fraction is in simplest form.

State The fraction of the student body that watches 0 to 2 hours of TV daily is $\frac{53}{60}$.

Check We can check by estimation. The result, $\frac{53}{60}$, is approximately $\frac{50}{60}$, which simplifies to $\frac{5}{6}$. The red, yellow, and blue shaded areas appear to shade about $\frac{5}{6}$ of the pie chart. The result seems reasonable.

ANSWERS TO SELF CHECKS

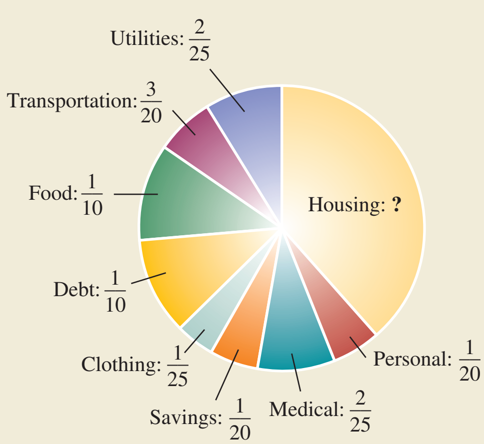
1. a. $\frac{1}{2}$ **b.** $\frac{7}{9}$ **2.** $-\frac{6}{11}$ **3.** $\frac{2}{3}$ **4.** $\frac{9}{10}$ **5.** $\frac{9}{35}$ **6.** $-\frac{3}{2}$ **7.** $-\frac{45}{8}$ **8.** $\frac{23}{24}$ **9.** $\frac{3}{20}$ **10.** $\frac{3}{5}$ **11.** $\frac{7}{12}$

THINK IT THROUGH *Budgets*

"Putting together a budget is crucial if you don't want to spend your way into serious problems. You're also developing a habit that can serve you well throughout your life."

Liz Pulliam Weston, MSN Money

The circle graph below shows a suggested budget for new college graduates as recommended by Springboard, a nonprofit consumer credit counseling service. What fraction of net take-home pay should be spent on housing?



SECTION 3.4 STUDY SET

VOCABULARY

Fill in the blanks.

1. Because the denominators of $\frac{3}{8}$ and $\frac{7}{8}$ are the same number, we say that they have a ________ denominator.
2. The ______ common denominator for a set of fractions is the smallest number each denominator will divide exactly (no remainder).
3. Consider the solution below. To ______ an equivalent fraction with a denominator of 18, we multiply $\frac{4}{9}$ by a 1 in the form of $\boxed{}$.

$$\frac{4}{9} = \frac{4}{9} \cdot \frac{2}{2}$$
$$= \frac{8}{18}$$

4. Consider the solution below. To ________ the fraction $\frac{15}{27}$, we factor 15 and 27, and then remove the common factor of 3 from the __________ and the __________.

$$\frac{15}{27} = \frac{\overset{1}{\cancel{3}} \cdot 5}{\underset{1}{\cancel{3}} \cdot 3 \cdot 3}$$
$$= \frac{5}{9}$$

CONCEPTS

Fill in the blanks.

5. To add (or subtract) fractions that have the same denominator, add (or subtract) their __________ and write the sum (or difference) over the _________ denominator. ________ the result, if possible.
6. To add (or subtract) fractions that have different denominators, we express each fraction as an equivalent fraction that has the ______ for its denominator. Then we use the rule for adding (subtracting) fractions that have the ______ denominator.
7. When adding (or subtracting) two fractions with different denominators, if the smaller denominator is a factor of the larger denominator, the _______ denominator is the LCD.
8. Write the subtraction as addition of the opposite:

$$-\frac{1}{8} - \left(-\frac{5}{8}\right) = \boxed{}\ \boxed{}\ \boxed{}$$

9. Consider $\frac{3}{4}$. By what form of 1 should we multiply the numerator and denominator to express it as an equivalent fraction with a denominator of 36?
10. The *denominators* of two fractions are given. Find the least common denominator.
    a. 2 and 3 **b.** 3 and 5
    c. 4 and 8 **d.** 6 and 36
11. Consider the following prime factorizations:

$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$
$$90 = 2 \cdot 3 \cdot 3 \cdot 5$$

For any one factorization, what is the greatest number of times

a. a 5 appears?
b. a 3 appears?
c. a 2 appears?

12. The *denominators* of two fractions have their prime-factored forms shown below. Fill in the blanks to find the LCD for the fractions.

$$\left.\begin{array}{l} 20 = 2 \cdot 2 \cdot 5 \\ 30 = 2 \cdot 3 \cdot 5 \end{array}\right\} \text{LCD} = \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} = \boxed{}$$

13. The *denominators* of three fractions have their prime-factored forms shown below. Fill in the blanks to find the LCD for the fractions.

$$\left.\begin{array}{l} 20 = 2 \cdot 2 \cdot 5 \\ 30 = 2 \cdot 3 \cdot 5 \\ 90 = 2 \cdot 3 \cdot 3 \cdot 5 \end{array}\right\} \text{LCD} = \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} = \boxed{}$$

14. Place a $>$ or $<$ symbol in the blank to make a true statement.

a. $\frac{32}{35}\ \boxed{}\ \frac{31}{35}$

b. $-\frac{13}{17}\ \boxed{}\ -\frac{11}{17}$

NOTATION

Fill in the blanks to complete each solution.

15. $\frac{2}{5} + \frac{1}{7} = \frac{2}{5} \cdot \frac{\square}{\square} + \frac{1}{7} \cdot \frac{5}{5}$

$= \frac{\square}{35} + \frac{5}{\square}$

$= \frac{\square + \square}{35}$

$= \frac{\square}{35}$

16. $\frac{7}{8} - \frac{2}{3} = \frac{7}{8} \cdot \frac{3}{3} - \frac{2}{3} \cdot \frac{\square}{\square}$

$= \frac{21}{\square} - \frac{16}{\square}$

$= \frac{21 - 16}{\square}$

$= \frac{\square}{24}$

GUIDED PRACTICE

Perform each operation and simplify, if possible. See Example 1.

17. $\frac{4}{9} + \frac{1}{9}$

18. $\frac{3}{7} + \frac{1}{7}$

19. $\frac{3}{8} + \frac{1}{8}$

20. $\frac{7}{12} + \frac{1}{12}$

21. $\frac{11}{15} - \frac{7}{15}$

22. $\frac{10}{21} - \frac{5}{21}$

23. $\frac{11}{20} - \frac{3}{20}$

24. $\frac{7}{18} - \frac{5}{18}$

Subtract and simplify, if possible. See Example 2.

25. $-\frac{11}{5} - \left(-\frac{8}{5}\right)$

26. $-\frac{15}{9} - \left(-\frac{11}{9}\right)$

27. $-\frac{7}{21} - \left(-\frac{2}{21}\right)$

28. $-\frac{21}{25} - \left(-\frac{9}{25}\right)$

Perform the operations and simplify, if possible. See Example 3.

29. $\frac{19}{40} - \frac{3}{40} - \frac{1}{40}$

30. $\frac{11}{24} - \frac{1}{24} - \frac{7}{24}$

31. $\frac{13}{33} + \frac{1}{33} + \frac{7}{33}$

32. $\frac{21}{50} + \frac{1}{50} + \frac{13}{50}$

Add and simplify, if possible. See Example 4.

33. $\frac{1}{3} + \frac{1}{7}$

34. $\frac{1}{4} + \frac{1}{5}$

35. $\frac{2}{5} + \frac{1}{2}$

36. $\frac{2}{7} + \frac{1}{2}$

Subtract and simplify, if possible. See Example 5.

37. $\frac{4}{5} - \frac{3}{4}$

38. $\frac{2}{3} - \frac{3}{5}$

39. $\frac{3}{4} - \frac{2}{7}$

40. $\frac{6}{7} - \frac{2}{3}$

Subtract and simplify, if possible. See Example 6.

41. $\frac{11}{12} - \frac{2}{3}$

42. $\frac{11}{18} - \frac{1}{6}$

43. $\frac{9}{14} - \frac{1}{7}$

44. $\frac{13}{15} - \frac{2}{3}$

Add and simplify, if possible. See Example 7.

45. $-2 + \frac{5}{9}$

46. $-3 + \frac{5}{8}$

47. $-3 + \frac{9}{4}$

48. $-1 + \frac{7}{10}$

Add and simplify, if possible. See Example 8.

49. $\frac{1}{6} + \frac{5}{8}$

50. $\frac{7}{12} + \frac{3}{8}$

51. $\frac{4}{9} + \frac{5}{12}$

52. $\frac{1}{9} + \frac{5}{6}$

Subtract and simplify, if possible. See Example 9.

53. $\frac{9}{10} - \frac{3}{14}$

54. $\frac{11}{12} - \frac{11}{30}$

55. $\frac{11}{12} - \frac{7}{15}$

56. $\frac{7}{15} - \frac{5}{12}$

Determine which fraction is larger. See Example 10.

57. $\frac{3}{8}$ or $\frac{5}{16}$

58. $\frac{5}{6}$ or $\frac{7}{12}$

59. $\frac{4}{5}$ or $\frac{2}{3}$

60. $\frac{7}{9}$ or $\frac{4}{5}$

61. $\frac{7}{9}$ or $\frac{11}{12}$

62. $\frac{3}{8}$ or $\frac{5}{12}$

63. $\frac{23}{20}$ or $\frac{7}{6}$

64. $\frac{19}{15}$ or $\frac{5}{4}$

Add and simplify, if possible. See Example 11.

65. $\frac{1}{6} + \frac{5}{18} + \frac{2}{9}$

66. $\frac{1}{10} + \frac{1}{8} + \frac{1}{5}$

67. $\frac{4}{15} + \frac{2}{3} + \frac{1}{6}$

68. $\frac{1}{2} + \frac{3}{5} + \frac{3}{20}$

TRY IT YOURSELF

Perform each operation.

69. $-\frac{1}{12} - \left(-\frac{5}{12}\right)$

70. $-\frac{1}{16} - \left(-\frac{15}{16}\right)$

71. $\frac{4}{5} + \frac{2}{3}$

72. $\frac{1}{4} + \frac{2}{3}$

73. $\frac{12}{25} - \frac{1}{25} - \frac{1}{25}$

74. $\frac{7}{9} + \frac{1}{9} + \frac{1}{9}$

75. $-\frac{7}{20} - \frac{1}{5}$

76. $-\frac{5}{8} - \frac{1}{3}$

77. $-\frac{7}{16} + \frac{1}{4}$

78. $-\frac{17}{20} + \frac{4}{5}$

79. $\frac{11}{12} - \frac{2}{3}$

80. $\frac{2}{3} - \frac{1}{6}$

81. $\frac{2}{3} + \frac{4}{5} + \frac{5}{6}$

82. $\frac{3}{4} + \frac{2}{5} + \frac{3}{10}$

83. $\frac{9}{20} - \frac{1}{30}$

84. $\frac{5}{6} - \frac{3}{10}$

85. $\frac{27}{50} + \frac{5}{16}$

86. $\frac{49}{50} - \frac{15}{16}$

87. $\frac{13}{20} - \frac{1}{5}$

88. $\frac{71}{100} - \frac{1}{10}$

89. $\frac{37}{103} - \frac{17}{103}$

90. $\frac{54}{53} - \frac{52}{53}$

91. $-\frac{3}{4} - 5$

92. $-2 - \frac{7}{8}$

93. $\frac{4}{27} + \frac{1}{6}$

94. $\frac{8}{9} - \frac{7}{12}$

95. $\frac{7}{30} - \frac{19}{75}$

96. $\frac{73}{75} - \frac{31}{30}$

97. Find the difference of $\frac{11}{60}$ and $\frac{2}{45}$.

98. Find the sum of $\frac{9}{48}$ and $\frac{7}{40}$.

99. Subtract $\frac{5}{12}$ from $\frac{2}{15}$.

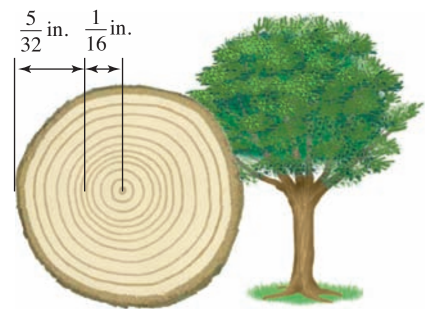
100. What is the sum of $\frac{11}{24}$ and $\frac{7}{36}$ increased by $\frac{5}{48}$?

APPLICATIONS

101. BOTANY To determine the effects of smog on tree development, a scientist cut down a pine tree and measured the width of the growth rings for the last two years.

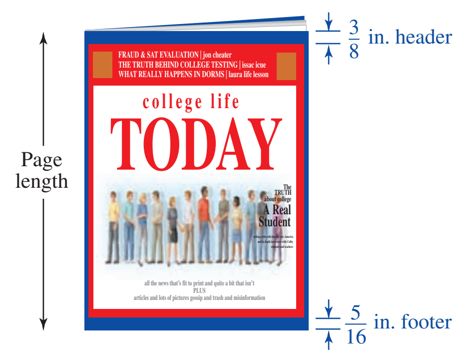
a. What was the growth over this two-year period?

b. What is the difference in the widths of the two rings?



102. GARAGE DOOR OPENERS What is the difference in strength between a $\frac{1}{3}$-hp and a $\frac{1}{2}$-hp garage door opener?

103. MAGAZINE COVERS The page design for the magazine cover shown below includes a blank strip at the top, called a *header*, and a blank strip at the bottom of the page, called a *footer*. How much page length is lost because of the header and footer?



104. DELIVERY TRUCKS A truck can safely carry a one-ton load. Should it be used to deliver one-half ton of sand, one-third ton of gravel, and one-fifth ton of cement in one trip to a job site?

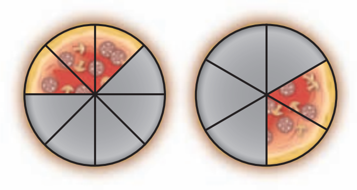
105. DINNERS A family bought two large pizzas for dinner. Some pieces of each pizza were not eaten, as shown.

a. What fraction of the first pizza was not eaten?

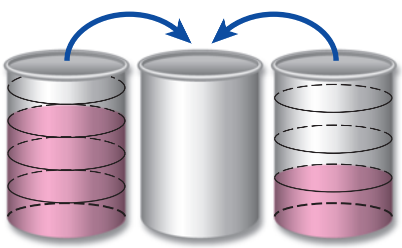
b. What fraction of the second pizza was not eaten?

c. What fraction of a pizza was left?

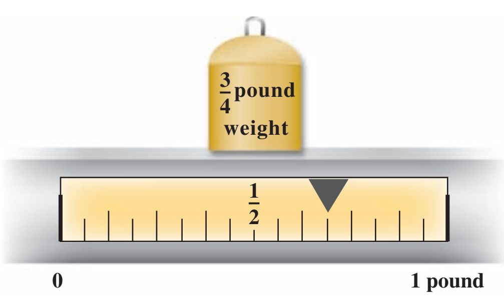
d. Could the family have been fed with just one pizza?



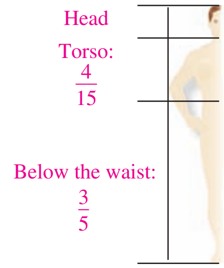
106. GASOLINE BARRELS Three identical-sized barrels are shown below. If their contents of the two of the barrels are poured into the empty third barrel, what fraction of the third barrel will be filled?



107. WEIGHTS AND MEASURES A consumer protection agency determines the accuracy of butcher shop scales by placing a known three-quarter-pound weight on the scale and then comparing that to the scale's readout. According to the illustration, by how much is this scale off? Does it result in undercharging or overcharging customers on their meat purchases?



108. FIGURE DRAWING As an aid in drawing the human body, artists divide the body into three parts. Each part is then expressed as a fraction of the total body height. For example, the torso is $\frac{4}{15}$ of the body height. What fraction of body height is the head?

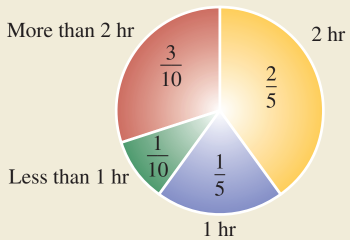


109. Suppose you work as a school guidance counselor at a community college and your department has conducted a survey of the full-time students to learn more about their study habits. As part of a *Power Point* presentation of the survey results to the school board, you show the following circle graph. At that time, you are asked, "What fraction of the full-time students study 2 hours or more daily?" What would you answer?

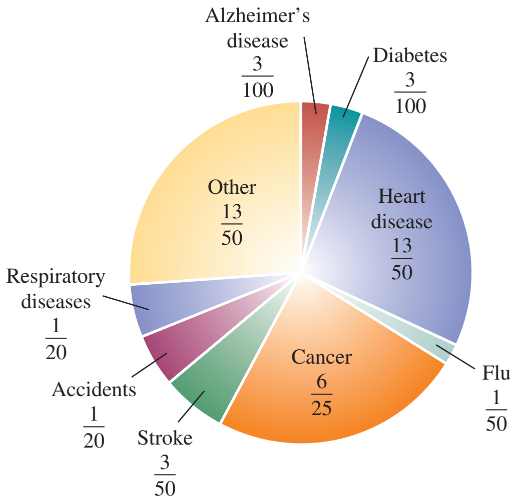
from Campus to Careers

School Guidance Counselor

iStockphoto.com/Monkeybusinessimages

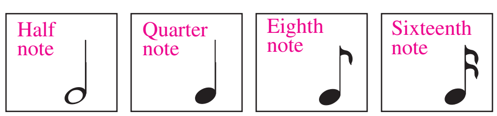


110. HEALTH STATISTICS The circle graph below shows the leading causes of death in the United States for 2006. For example, $\frac{13}{50}$ of all of the deaths that year were caused by heart disease. What fraction of all the deaths were caused by heart disease, cancer, or stroke, combined?

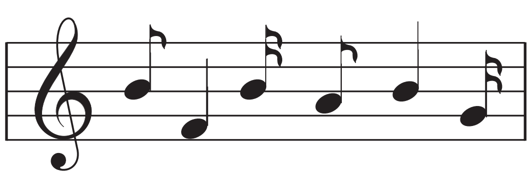


Source: National Center for Health Statistics

111. MUSICAL NOTES The notes used in music have fractional values. Their names and the symbols used to represent them are shown in illustration (a). In common time, the values of the notes in each measure must add to 1. Is the measure in illustration (b) complete?



(a)



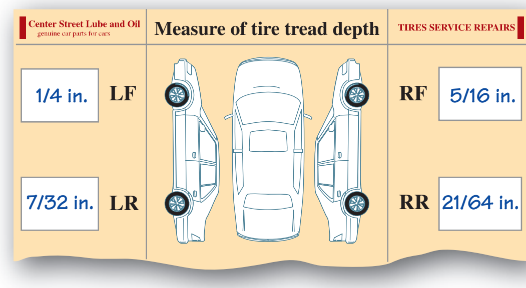
(b)

112. TOOLS A mechanic likes to hang his wrenches above his tool bench in order of narrowest to widest. What is the proper order of the wrenches in the illustration?

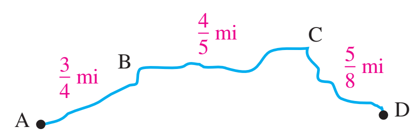
113. TIRE TREAD A mechanic measured the tire tread depth on each of the tires on a car and recorded them on the form shown below. (The letters LF stand for *left front*, RR stands for *right rear,* and so on.)

a. Which tire has the most tread?

b. Which tire has the least tread?



114. HIKING The illustration below shows the length of each part of a three-part hike. Rank the lengths of the parts from longest to shortest.



WRITING

115. Explain why we cannot add or subtract the fractions $\frac{2}{9}$ and $\frac{2}{5}$ as they are written.

116. To multiply fractions, must they have the same denominators? Explain why or why not. Give an example.

REVIEW

Perform each operation and simplify, if possible.

117. a. $\frac{1}{4} + \frac{1}{8}$ **b.** $\frac{1}{4} - \frac{1}{8}$

c. $\frac{1}{4} \cdot \frac{1}{8}$ **d.** $\frac{1}{4} \div \frac{1}{8}$

118. a. $\frac{5}{21} + \frac{3}{14}$ **b.** $\frac{5}{21} - \frac{3}{14}$

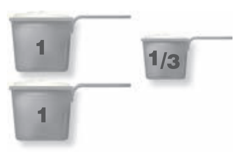
c. $\frac{5}{21} \cdot \frac{3}{14}$ **d.** $\frac{5}{21} \div \frac{3}{14}$

SECTION 3.5
Multiplying and Dividing Mixed Numbers

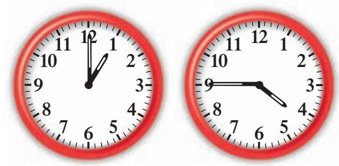
Objectives

1. Identify the whole-number and fractional parts of a mixed number.
2. Write mixed numbers as improper fractions.
3. Write improper fractions as mixed numbers.
4. Graph fractions and mixed numbers on a number line.
5. Multiply and divide mixed numbers.
6. Solve application problems by multiplying and dividing mixed numbers.

In the next two sections, we show how to add, subtract, multiply, and divide *mixed numbers.* These numbers are widely used in daily life.



The recipe calls for $2\frac{1}{3}$ cups of flour.
(Read as "two and one-third.")



It took $3\frac{3}{4}$ hours to paint the living room.
(Read as "three and three-fourths.")

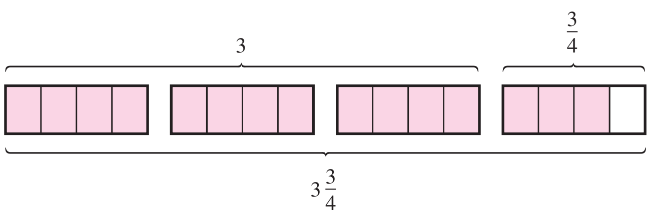
The entrance to the park is $1\frac{1}{2}$ miles away.
(Read as "one and one-half.")

1 Identify the whole-number and fractional parts of a mixed number.

A **mixed number** is the *sum* of a whole number and a proper fraction. For example, $3\frac{3}{4}$ is a mixed number.

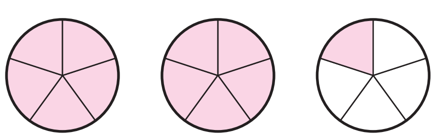
$$\underset{\text{Mixed number}}{3\frac{3}{4}} = \underset{\text{Whole-number part}}{3} + \underset{\text{Fractional part}}{\frac{3}{4}}$$

Mixed numbers can be represented by shaded regions. In the illustration below, each rectangular region outlined in black represents one whole. To represent $3\frac{3}{4}$, we shade 3 *whole* rectangular regions and 3 out of 4 *parts* of another.



Caution! Note that $3\frac{3}{4}$ means $3 + \frac{3}{4}$, even though the + symbol is not written. Do not confuse $3\frac{3}{4}$ with $3 \cdot \frac{3}{4}$ or $3\left(\frac{3}{4}\right)$, which indicate the multiplication of 3 by $\frac{3}{4}$.

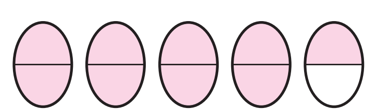
EXAMPLE 1 In the illustration below, each disk represents one whole. Write an improper fraction and a mixed number to represent the shaded portion.



Strategy We will determine the number of equal parts into which a disk is divided. Then we will determine how many of those *parts* are shaded and how many of the *whole* disks are shaded.

Self Check 1

In the illustration below, each oval region represents one whole. Write an improper fraction and a mixed number to represent the shaded portion.



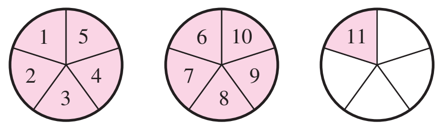
Now Try **Problem 19**

WHY To write an improper fraction, we need to find its numerator and its denominator. To write a mixed number, we need to find its whole number part and its fractional part.

Solution

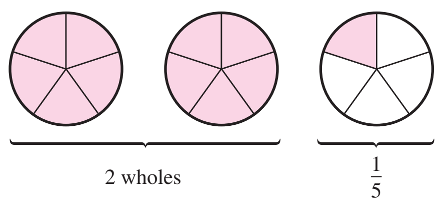
Since each disk is divided into 5 equal parts, the denominator of the improper fraction is 5. Since a total of 11 of those parts are shaded, the numerator is 11, and we say that

$\frac{11}{5}$ is shaded. Write: $\frac{\text{total number of parts shaded}}{\text{number of equal parts in one disk}}$



Since 2 whole disks are shaded, the whole number part of the mixed number is 2. Since 1 out of 5 of the parts of the last disk is shaded, the fractional part of the mixed number is $\frac{1}{5}$, and we say that

$2\frac{1}{5}$ is shaded.



In this section, we will work with negative as well as positive mixed numbers. For example, the negative mixed number $-3\frac{3}{4}$ could be used to represent $3\frac{3}{4}$ feet below sea level. Think of $-3\frac{3}{4}$ as $-3 - \frac{3}{4}$ or as $-3 + \left(-\frac{3}{4}\right)$.

2 Write mixed numbers as improper fractions.

In Example 1, we saw that the shaded portion of the illustration can be represented by the mixed number $2\frac{1}{5}$ and by the improper fraction $\frac{11}{5}$. To develop a procedure to write any mixed number as an improper fraction, consider the following steps that show how to do this for $2\frac{1}{5}$. The objective is to find how many *fifths* that the mixed number $2\frac{1}{5}$ represents.

$$2\frac{1}{5} = 2 + \frac{1}{5} \qquad \text{Write the mixed number } 2\tfrac{1}{5} \text{ as a sum.}$$

$$= \frac{2}{1} + \frac{1}{5} \qquad \text{Write 2 as a fraction: } 2 = \tfrac{2}{1}.$$

$$= \frac{2}{1} \cdot \frac{5}{5} + \frac{1}{5} \qquad \text{To build } \tfrac{2}{1} \text{ so that its denominator is 5, multiply it by a form of 1.}$$

$$= \frac{10}{5} + \frac{1}{5} \qquad \begin{array}{l}\text{Multiply the numerators.}\\ \text{Multiply the denominators.}\end{array}$$

$$= \frac{11}{5} \qquad \begin{array}{l}\text{Add the numerators and write the sum over}\\ \text{the common denominator 5.}\end{array}$$

Thus, $2\frac{1}{5} = \frac{11}{5}$.

We can obtain the same result with far less work. To change $2\frac{1}{5}$ to an improper fraction, we simply multiply 5 by 2 and add 1 to get the numerator, and keep the denominator of 5.

$$2\frac{1}{5} = \frac{5 \cdot 2 + 1}{5} = \frac{10 + 1}{5} = \frac{11}{5}$$

This example illustrates the following procedure.

Writing a Mixed Number as an Improper Fraction

To write a mixed number as an improper fraction:

1. Multiply the denominator of the fraction by the whole-number part.
2. Add the numerator of the fraction to the result from Step 1.
3. Write the sum from Step 2 over the original denominator.

EXAMPLE 2 Write the mixed number $7\frac{5}{6}$ as an improper fraction.

Strategy We will use the 3-step procedure to find the improper fraction.

WHY It's faster than writing $7\frac{5}{6}$ as $7 + \frac{5}{6}$, building to get an LCD, and adding.

Solution

To find the numerator of the improper fraction, multiply 6 by 7, and add 5 to that result. The denominator of the improper fraction is the same as the denominator of the fractional part of the mixed number.

Step 2: add

$$7\frac{5}{6} = \frac{6 \cdot 7 + 5}{6} = \frac{42 + 5}{6} = \frac{47}{6}$$

Step 1: multiply

Step 3: Use the same denominator

By the order of operations rule, multiply first, and then add in the numerator.

Self Check 2

Write the mixed number $3\frac{3}{8}$ as an improper fraction.

Now Try **Problems 23 and 27**

To write a *negative mixed number* in fractional form, ignore the − sign and use the method shown in Example 2 on the positive mixed number. Once that procedure is completed, write a − sign in front of the result. For example,

$$-6\frac{1}{4} = -\frac{25}{4} \qquad -1\frac{9}{10} = -\frac{19}{10} \qquad -12\frac{3}{8} = -\frac{99}{8}$$

3 Write improper fractions as mixed numbers.

To write an improper fraction as a mixed number, we must find two things: the *whole-number part* and the *fractional part* of the mixed number. To develop a procedure to do this, let's consider the improper fraction $\frac{7}{3}$. To find the number of groups of 3 in 7, we can divide 7 by 3. This will find the whole-number part of the mixed number. The remainder is the numerator of the fractional part of the mixed number.

Whole-number part

$$\begin{array}{r} 2 \\ 3\overline{)7} \\ \underline{-6} \\ 1 \end{array} \qquad 2\frac{1}{3}$$

The remainder is the numerator of the fractional part.

The divisor is the denominator of the fractional part.

This example suggests the following procedure.

Writing an Improper Fraction as a Mixed Number

To write an improper fraction as a mixed number:

1. Divide the numerator by the denominator to obtain the whole-number part.
2. The remainder over the divisor is the fractional part.

Self Check 3

Write each improper fraction as a mixed number or a whole number:

a. $\frac{31}{7}$ **b.** $\frac{50}{26}$

c. $\frac{51}{3}$ **d.** $-\frac{10}{3}$

Now Try **Problems 31, 35, 39, and 43**

EXAMPLE 3 Write each improper fraction as a mixed number or a whole number: **a.** $\frac{29}{6}$ **b.** $\frac{40}{16}$ **c.** $\frac{84}{3}$ **d.** $-\frac{9}{5}$

Strategy We will divide the numerator by the denominator and write the remainder over the divisor.

WHY A fraction bar indicates division.

Solution

a. To write $\frac{29}{6}$ as a mixed number, divide 29 by 6:

$$\begin{array}{r} 4 \\ 6\overline{)29} \\ -24 \\ \hline 5 \end{array}$$

4 ← The whole-number part is 4.

5 ← Write the remainder 5 over the divisor 6 to get the fractional part.

Thus, $\frac{29}{6} = 4\frac{5}{6}$.

b. To write $\frac{40}{16}$ as a mixed number, divide 40 by 16:

$$\begin{array}{r} 2 \\ 16\overline{)40} \\ -32 \\ \hline 8 \end{array}$$

Thus, $\frac{40}{16} = 2\frac{8}{16} = 2\frac{1}{2}$. Simplify the fractional part: $\frac{8}{16} = \frac{\cancel{8}^{1}}{2 \cdot \cancel{8}_{1}} = \frac{1}{2}$.

c. For $\frac{84}{3}$, divide 84 by 3:

$$\begin{array}{r} 28 \\ 3\overline{)84} \\ -6 \\ \hline 24 \\ -24 \\ \hline 0 \end{array}$$

Thus, $\frac{84}{3} = 28$.

0 ← Since the remainder is 0, the improper fraction represents a whole number.

d. To write $-\frac{9}{5}$ as a mixed number, ignore the – sign, and use the method for the positive improper fraction $\frac{9}{5}$. Once that procedure is completed, write a – sign in front of the result.

$$\begin{array}{r} 1 \\ 5\overline{)9} \\ -5 \\ \hline 4 \end{array}$$

Thus, $-\frac{9}{5} = -1\frac{4}{5}$.

4 Graph fractions and mixed numbers on a number line.

In Chapters 1 and 2, we graphed whole numbers and integers on a number line. Fractions and mixed numbers can also be graphed on a number line.

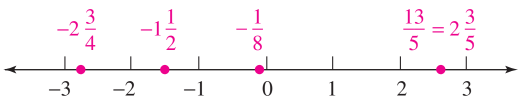
EXAMPLE 4 Graph $-2\frac{3}{4}, -1\frac{1}{2}, -\frac{1}{8}$, and $\frac{13}{5}$ on a number line.

Strategy We will locate the position of each fraction and mixed number on the number line and draw a bold dot.

WHY To *graph a number* means to make a drawing that represents the number.

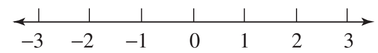
Solution

- Since $-2\frac{3}{4} < -2$, the graph of $-2\frac{3}{4}$ is to the left of -2 on the number line.
- The number $-1\frac{1}{2}$ is between -1 and -2.
- The number $-\frac{1}{8}$ is less than 0.
- Expressed as a mixed number, $\frac{13}{5} = 2\frac{3}{5}$.



Self Check 4

Graph $-1\frac{7}{8}, -\frac{2}{3}, \frac{3}{5}$, and $\frac{9}{4}$ on a number line.



Now Try **Problem 47**

5 Multiply and divide mixed numbers.

We will use the same procedures for multiplying and dividing mixed numbers as those that were used in Sections 3.2 and 3.3 to multiply and divide fractions. However, we must write the mixed numbers as improper fractions before we actually multiply or divide.

Multiplying and Dividing Mixed Numbers

To multiply or divide mixed numbers, first change the mixed numbers to improper fractions. Then perform the multiplication or division of the fractions. Write the result as a mixed number or a whole number in simplest form.

The sign rules for multiplying and dividing integers also hold for multiplying and dividing mixed numbers.

EXAMPLE 5 Multiply and simplify, if possible.

a. $1\frac{3}{4} \cdot 2\frac{1}{3}$ **b.** $5\frac{1}{5} \cdot \left(1\frac{2}{13}\right)$ **c.** $-4\frac{1}{9}(3)$

Strategy We will write the mixed numbers and whole numbers as improper fractions.

WHY Then we can use the rule for multiplying two fractions from Section 3.2.

Solution

a. $1\frac{3}{4} \cdot 2\frac{1}{3} = \frac{7}{4} \cdot \frac{7}{3}$ Write $1\frac{3}{4}$ and $2\frac{1}{3}$ as improper fractions.

$= \frac{7 \cdot 7}{4 \cdot 3}$ Use the rule for multiplying two fractions. Multiply the numerators and the denominators.

$= \frac{49}{12}$ Since there are no common factors to remove, perform the multiplication in the numerator and in the denominator. The result is an improper fraction.

$= 4\frac{1}{12}$ Write the improper fraction $\frac{49}{12}$ as a mixed number.

$$\begin{array}{r} 4 \\ 12\overline{)49} \\ -48 \\ \hline 1 \end{array}$$

Self Check 5

Multiply and simplify, if possible.

a. $3\frac{1}{3} \cdot 2\frac{1}{3}$ **b.** $9\frac{3}{5} \cdot \left(3\frac{3}{4}\right)$

c. $-4\frac{5}{6}(2)$

Now Try **Problems 51, 55, and 57**

b. $5\frac{1}{5}\left(1\frac{2}{13}\right) = \frac{26}{5} \cdot \frac{15}{13}$ Write $5\frac{1}{5}$ and $1\frac{2}{13}$ as improper fractions.

$= \frac{26 \cdot 15}{5 \cdot 13}$ Multiply the numerators. Multiply the denominators.

$= \frac{2 \cdot 13 \cdot 3 \cdot 5}{5 \cdot 13}$ To prepare to simplify, factor 26 as $2 \cdot 13$ and 15 as $3 \cdot 5$.

$= \frac{2 \cdot \overset{1}{\cancel{13}} \cdot 3 \cdot \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{13}}}$ Remove the common factors of 13 and 5 from the numerator and denominator.

$= \frac{6}{1}$ Multiply the remaining factors in the numerator: $2 \cdot 1 \cdot 3 \cdot 1 = 6$. Multiply the remaining factors in the denominator: $1 \cdot 1 = 1$.

$= 6$ Any whole number divided by 1 remains the same.

c. $-4\frac{1}{9} \cdot 3 = -\frac{37}{9} \cdot \frac{3}{1}$ Write $-4\frac{1}{9}$ as an improper fraction and write 3 as a fraction.

$= -\frac{37 \cdot 3}{9 \cdot 1}$ Multiply the numerators and multiply the denominators. Since the fractions have unlike signs, make the answer negative.

$= -\frac{37 \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \cdot 3 \cdot 1}$ To simplify, factor 9 as $3 \cdot 3$, and then remove the common factor of 3 from the numerator and denominator.

$= -\frac{37}{3}$ Multiply the remaining factors in the numerator and in the denominator. The result is an improper fraction.

$= -12\frac{1}{3}$ Write the negative improper fraction $-\frac{37}{3}$ as a negative mixed number.

$$\begin{array}{r} 12 \\ 3\overline{)37} \\ \underline{-3} \\ 7 \\ \underline{-6} \\ 1 \end{array}$$

Success Tip We can use rounding to check the results when multiplying mixed numbers. If the fractional part of the mixed number is $\frac{1}{2}$ *or greater,* round up by adding 1 to the whole-number part and dropping the fraction. If the fractional part of the mixed number is less than $\frac{1}{2}$, round down by dropping the fraction and using only the whole-number part. To check the answer $4\frac{1}{12}$ from Example 5, part a, we proceed as follows:

$1\frac{3}{4} \cdot 2\frac{1}{3} \approx 2 \cdot 2 = 4$ Since $\frac{3}{4}$ is greater than $\frac{1}{2}$, round $1\frac{3}{4}$ up to 2. Since $\frac{1}{3}$ is less than $\frac{1}{2}$, round $2\frac{1}{3}$ down to 2.

Since $4\frac{1}{12}$ is close to 4, it is a reasonable answer.

Self Check 6

Divide and simplify, if possible:

a. $-3\frac{4}{15} \div \left(-2\frac{1}{10}\right)$

b. $5\frac{3}{5} \div \frac{7}{8}$

Now Try **Problems 59 and 65**

EXAMPLE 6 Divide and simplify, if possible:

a. $-3\frac{3}{8} \div \left(-2\frac{1}{4}\right)$ **b.** $1\frac{11}{16} \div \frac{3}{4}$

Strategy We will write the mixed numbers as improper fractions.

WHY Then we can use the rule for dividing two fractions from Section 3.3.

Solution

a. $-3\frac{3}{8} \div \left(-2\frac{1}{4}\right) = -\frac{27}{8} \div \left(-\frac{9}{4}\right)$ Write $-3\frac{3}{8}$ and $-2\frac{1}{4}$ as improper fractions.

$= -\frac{27}{8}\left(-\frac{4}{9}\right)$ Use the rule for dividing two fractions.: Multiply $-\frac{27}{8}$ by the reciprocal of $-\frac{9}{4}$, which is $-\frac{4}{9}$.

$$= \frac{27}{8}\left(\frac{4}{9}\right)$$ Since the product of two negative fractions is positive, drop both − signs and continue.

$$= \frac{27 \cdot 4}{8 \cdot 9}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{3 \cdot \overset{1}{\cancel{9}} \cdot \overset{1}{\cancel{4}}}{2 \cdot \underset{1}{\cancel{4}} \cdot \underset{1}{\cancel{9}}}$$ To simplify, factor 27 as $3 \cdot 9$ and 8 as $2 \cdot 4$. Then remove the common factors of 9 and 4 from the numerator and denominator.

$$= \frac{3}{2}$$ Multiply the remaining factors in the numerator: $3 \cdot 1 \cdot 1 = 3$. Multiply the remaining factors in the denominator: $2 \cdot 1 \cdot 1 = 2$.

$$= 1\frac{1}{2}$$ Write the improper fraction $\frac{3}{2}$ as a mixed number by dividing 3 by 2.

b. $$1\frac{11}{16} \div \frac{3}{4} = \frac{27}{16} \div \frac{3}{4}$$ Write $1\frac{11}{16}$ as an improper fraction.

$$= \frac{27}{16} \cdot \frac{4}{3}$$ Multiply $\frac{27}{16}$ by the reciprocal of $\frac{3}{4}$, which is $\frac{4}{3}$.

$$= \frac{27 \cdot 4}{16 \cdot 3}$$ Multiply the numerators. Multiply the denominators.

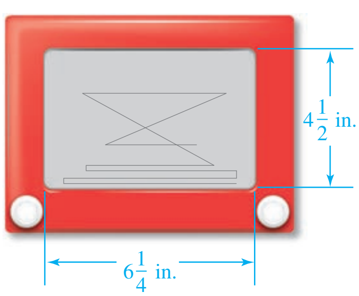
$$= \frac{\overset{1}{\cancel{3}} \cdot 9 \cdot \overset{1}{\cancel{4}}}{\underset{1}{\cancel{4}} \cdot 4 \cdot \underset{1}{\cancel{3}}}$$ To simplify, factor 27 as $3 \cdot 9$ and 16 as $4 \cdot 4$. Then remove the common factors of 3 and 4 from the numerator and denominator.

$$= \frac{9}{4}$$ Multiply the remaining factors in the numerator and in the denominator. The result is an improper fraction.

$$= 2\frac{1}{4}$$ Write the improper fraction $\frac{9}{4}$ as a mixed number by dividing 9 by 4.

6 Solve application problems by multiplying and dividing mixed numbers.

EXAMPLE 7 *Toys* The dimensions of the rectangular-shaped screen of an Etch-a-Sketch are shown in the illustration below. Find the area of the screen.



Strategy To find the area, we will multiply $6\frac{1}{4}$ by $4\frac{1}{2}$.

WHY The formula for the area of a rectangle is Area = length · width.

Self Check 7

BUMPER STICKERS A rectangular-shaped bumper sticker is $8\frac{1}{4}$ inches long by $3\frac{1}{4}$ inches wide. Find its area.

Now Try **Problem 99**

Solution

$A = lw$ This is the formula for the area of a rectangle.

$= 6\frac{1}{4} \cdot 4\frac{1}{2}$ Substitute $6\frac{1}{4}$ for l and $4\frac{1}{2}$ for w.

$= \frac{25}{4} \cdot \frac{9}{2}$ Write $6\frac{1}{4}$ and $4\frac{1}{2}$ as improper fractions.

$= \frac{25 \cdot 9}{4 \cdot 2}$ Multiply the numerators. Multiply the denominators.

$= \frac{225}{8}$ Since there are no common factors to remove, perform the multiplication in the numerator and in the denominator. The result is an improper fraction.

$= 28\frac{1}{8}$ Write the improper fraction $\frac{225}{8}$ as a mixed number.

$$\begin{array}{r} 28 \\ 8\overline{)225} \\ -16 \\ \hline 65 \\ -64 \\ \hline 1 \end{array}$$

The area of the screen of an Etch-a-Sketch is $28\frac{1}{8}$ in.^{2}.

Self Check 8

TV INTERVIEWS An $18\frac{3}{4}$-minute taped interview with an actor was played in equally long segments over 5 consecutive nights on a celebrity news program. How long was each interview segment?

Now Try **Problem 107**

EXAMPLE 8 *Government Grants*

If $\$12\frac{1}{2}$ million is to be split equally among five cities to fund recreation programs, how much will each city receive?

Analyze

- There is $\$12\frac{1}{2}$ million in grant money. *Given*
- 5 cities will split the money equally. *Given*
- How much grant money will each city receive? *Find*

Form

The key phrase *split equally* suggests division.

We translate the words of the problem to numbers and symbols.

The amount of money that each city will receive (in millions of dollars)	is equal to	the total amount of grant money (in millions of dollars)	divided by	the number of cities receiving money.
The amount of money that each city will receive (in millions of dollars)	$=$	$12\frac{1}{2}$	$\div$	5

Solve

To find the quotient, we will express $12\frac{1}{2}$ and 5 as fractions and then use the rule for dividing two fractions.

$12\frac{1}{2} \div 5 = \frac{25}{2} \div \frac{5}{1}$ Write $12\frac{1}{2}$ as an improper fraction, and write 5 as a fraction.

$= \frac{25}{2} \cdot \frac{1}{5}$ Multiply by the reciprocal of $\frac{5}{1}$, which is $\frac{1}{5}$.

$= \frac{25 \cdot 1}{2 \cdot 5}$ Multiply the numerators. Multiply the denominators.

$= \frac{\overset{1}{\cancel{5}} \cdot 5 \cdot 1}{2 \cdot \underset{1}{\cancel{5}}}$ To simplify, factor 25 as $5 \cdot 5$. Then remove the common factor of 5 from the numerator and denominator.

$= \frac{5}{2}$ Multiply the remaining factors in the numerator. Multiply the remaining factors in the denominator.

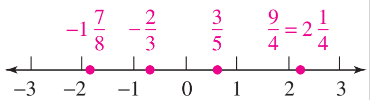
$= 2\frac{1}{2}$ Write the improper fraction $\frac{5}{2}$ as a mixed number by dividing 5 by 2. The units are in millions of dollars.

State Each city will receive $\$2\frac{1}{2}$ million in grant money.

Check We can estimate to check the result. If there was \$10 million in grant money, each city would receive $\frac{\$10 \text{ million}}{5}$, or \$2 million. Since there is actually $\$12\frac{1}{2}$ million in grant money, the answer that each city would receive $\$2\frac{1}{2}$ million seems reasonable.

ANSWERS TO SELF CHECKS

1. $\frac{9}{2}$, $4\frac{1}{2}$ **2.** $\frac{27}{8}$ **3. a.** $4\frac{3}{7}$ **b.** $1\frac{12}{13}$ **c.** 17 **d.** $-3\frac{1}{3}$ **4.**



5. a. $7\frac{7}{9}$ **b.** 36 **c.** $-9\frac{2}{3}$ **6. a.** $1\frac{5}{9}$ **b.** $6\frac{2}{5}$ **7.** $26\frac{13}{16}$ in.^{2} **8.** $3\frac{3}{4}$ min

SECTION 3.5 STUDY SET

VOCABULARY

Fill in the blanks.

1. A _______ number, such as $8\frac{4}{5}$, is the sum of a whole number and a proper fraction.

2. In the mixed number $8\frac{4}{5}$, the _______-number part is 8 and the _________ part is $\frac{4}{5}$.

3. The numerator of an _________ fraction is greater than or equal to its denominator.

4. To ______ a number means to locate its position on the number line and highlight it using a dot.

CONCEPTS

5. What signed mixed number could be used to describe each situation?

a. A temperature of five and one-third degrees above zero

b. The depth of a sprinkler pipe that is six and seven-eighths inches below the sidewalk

6. What signed mixed number could be used to describe each situation?

a. A rain total two and three-tenths of an inch lower than the average

b. Three and one-half minutes after the liftoff of a rocket

Fill in the blanks.

7. To write a mixed number as an improper fraction:

1. ________ the denominator of the fraction by the whole-number part.

2. _____ the numerator of the fraction to the result from Step 1.

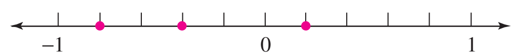
3. Write the sum from Step 2 over the original ___________.

8. To write an improper fraction as a mixed number:

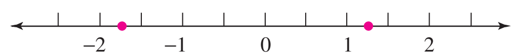
1. _______ the numerator by the denominator to obtain the whole-number part.

2. The __________ over the divisor is the fractional part.

9. What fractions have been graphed on the number line?



10. What mixed numbers have been graphed on the number line?



11. Fill in the blank: To multiply or divide mixed numbers, first change the mixed numbers to _________ fractions. Then perform the multiplication or division of the fractions as usual.

12. Simplify the fractional part of each mixed number.

a. $11\frac{2}{4}$

b. $1\frac{3}{9}$

c. $7\frac{15}{27}$

13. Use *estimation* to determine whether the following answer seems reasonable:

$$4\frac{1}{5} \cdot 2\frac{5}{7} = 7\frac{2}{35}$$

14. What is the formula for the

a. area of a rectangle?

b. area of a triangle?

NOTATION

15. Fill in the blanks.

a. We read $5\frac{11}{16}$ as "five _____ eleven-__________."

b. We read $-4\frac{2}{3}$ as "________ four and _____-thirds."

16. Determine the sign of the result. *You do not have to find the answer.*

a. $1\frac{1}{9}\left(-7\frac{3}{14}\right)$

b. $-3\frac{4}{15} \div \left(-1\frac{5}{6}\right)$

Fill in the blanks to complete each solution.

17. Multiply: $5\frac{1}{4} \cdot 1\frac{1}{7}$

$$5\frac{1}{4} \cdot 1\frac{1}{7} = \frac{21}{\square} \cdot \frac{\square}{7}$$
$$= \frac{21 \cdot \square}{\square \cdot 7}$$
$$= \frac{3 \cdot \overset{1}{\cancel{7}} \cdot 2 \cdot \overset{1}{\cancel{\square}}}{\underset{1}{\cancel{\square}} \cdot \underset{1}{\cancel{7}}}$$
$$= \frac{\square}{1}$$
$$= \square$$

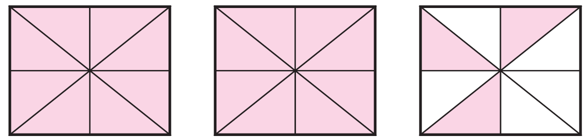
18. Divide: $-5\frac{5}{6} \div 2\frac{1}{12}$

$$-5\frac{5}{6} \div 2\frac{1}{12} = -\frac{\square}{6} \div \frac{25}{\square}$$
$$= -\frac{\square}{6} \cdot \frac{12}{\square}$$
$$= -\frac{35 \cdot 12}{6 \cdot \square}$$
$$= -\frac{\overset{1}{\cancel{5}} \cdot \square \cdot 2 \cdot \overset{1}{\cancel{6}}}{\underset{1}{\cancel{6}} \cdot \underset{1}{\cancel{5}} \cdot \square}$$
$$= -\frac{\square}{5}$$
$$= -2\frac{\square}{5}$$

GUIDED PRACTICE

Each region outlined in black represents one whole. Write an improper fraction and a mixed number to represent the shaded portion. See Example 1.

19.



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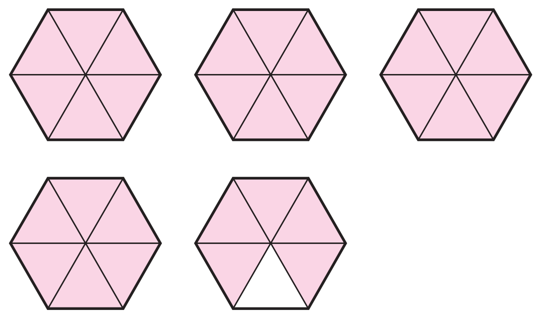

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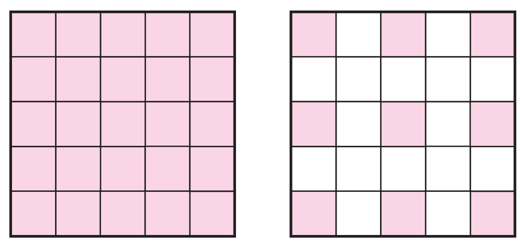

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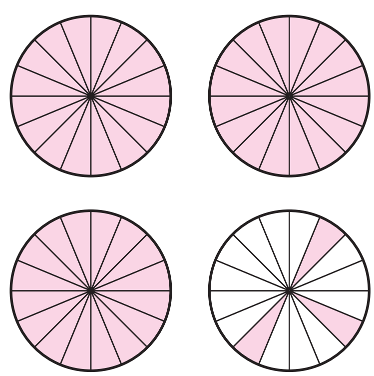


Write each mixed number as an improper fraction. See Example 2.

23. $6\frac{1}{2}$

24. $8\frac{2}{3}$

25. $20\frac{4}{5}$

26. $15\frac{3}{8}$

27. $-7\frac{5}{9}$

28. $-7\frac{1}{12}$

29. $-8\frac{2}{3}$

30. $-9\frac{3}{4}$

Write each improper fraction as a mixed number or a whole number. Simplify the result, if possible. See Example 3.

31. $\frac{13}{4}$

32. $\frac{41}{6}$

33. $\frac{28}{5}$

34. $\frac{28}{3}$

35. $\frac{42}{9}$

36. $\frac{62}{8}$

37. $\frac{84}{8}$

38. $\frac{93}{9}$

39. $\frac{52}{13}$

40. $\frac{80}{16}$

41. $\frac{34}{17}$

42. $\frac{38}{19}$

43. $-\frac{58}{7}$

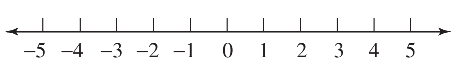
44. $-\frac{33}{7}$

45. $-\frac{20}{6}$

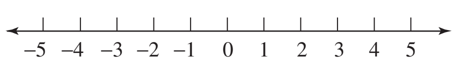
46. $-\frac{28}{8}$

Graph the given numbers on a number line. See Example 4.

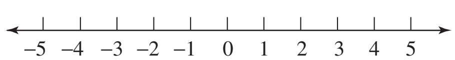
47. $-2\frac{8}{9}, 1\frac{2}{3}, \frac{16}{5}, -\frac{1}{2}$



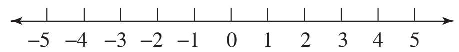
48. $-\frac{3}{4}, -3\frac{1}{4}, \frac{5}{2}, 4\frac{3}{4}$



49. $3\frac{1}{7}, -\frac{98}{99}, -\frac{10}{3}, \frac{3}{2}$



50. $-2\frac{1}{5}, \frac{4}{5}, -\frac{11}{3}, \frac{17}{4}$



Multiply and simplify, if possible. See Example 5.

51. $3\frac{1}{2} \cdot 2\frac{1}{3}$

52. $1\frac{5}{6} \cdot 1\frac{1}{2}$

53. $2\frac{2}{5}\left(3\frac{1}{12}\right)$

54. $\frac{40}{16}\left(\frac{26}{5}\right)$

55. $6\frac{1}{2} \cdot 1\frac{3}{13}$

56. $12\frac{3}{5} \cdot 1\frac{3}{7}$

57. $-2\frac{1}{2}(4)$

58. $-3\frac{3}{4}(8)$

Divide and simplify, if possible. See Example 6.

59. $-1\frac{13}{15} \div \left(-4\frac{1}{5}\right)$

60. $-2\frac{5}{6} \div \left(-8\frac{1}{2}\right)$

61. $15\frac{1}{3} \div 2\frac{2}{9}$

62. $6\frac{1}{4} \div 3\frac{3}{4}$

63. $1\frac{3}{4} \div \frac{3}{4}$

64. $5\frac{3}{5} \div \frac{9}{10}$

65. $1\frac{7}{24} \div \frac{7}{8}$

66. $4\frac{1}{2} \div \frac{3}{17}$

TRY IT YOURSELF

Perform each operation and simplify, if possible.

67. $-6 \cdot 2\frac{7}{24}$

68. $-7 \cdot 1\frac{3}{28}$

69. $-6\frac{3}{5} \div 7\frac{1}{3}$

70. $-4\frac{1}{4} \div 4\frac{1}{2}$

71. $\left(1\frac{2}{3}\right)^2$

72. $\left(3\frac{1}{2}\right)^2$

73. $8 \div 3\frac{1}{5}$

74. $15 \div 3\frac{1}{3}$

75. $-20\frac{1}{4} \div \left(-1\frac{11}{16}\right)$

76. $-2\frac{7}{10} \div \left(-1\frac{1}{14}\right)$

77. $3\frac{1}{16} \cdot 4\frac{4}{7}$

78. $5\frac{3}{5} \cdot 1\frac{11}{14}$

79. Find the quotient of $-4\frac{1}{2}$ and $2\frac{1}{4}$.

80. Find the quotient of 25 and $-10\frac{5}{7}$.

81. $2\frac{1}{2}\left(-3\frac{1}{3}\right)$

82. $\left(-3\frac{1}{4}\right)\left(1\frac{1}{5}\right)$

83. $2\frac{5}{8} \cdot \frac{5}{27}$

84. $3\frac{1}{9} \cdot \frac{3}{32}$

85. $6\frac{1}{4} \div 20$

86. $4\frac{2}{5} \div 11$

87. Find the product of $1\frac{2}{3}$, 6, and $-\frac{1}{8}$.

88. Find the product of $-\frac{5}{6}$, -8, and $-2\frac{1}{10}$.

89. $\left(-1\frac{1}{3}\right)^3$

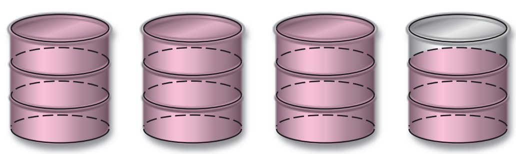
90. $\left(-1\frac{1}{5}\right)^3$

APPLICATIONS

91. In the illustration below, each barrel represents one whole.

a. Write a mixed number to represent the shaded portion.

b. Write an improper fraction to represent the shaded portion.



92. Draw $\frac{17}{8}$ pizzas.

93. DIVING Fill in the blank with a mixed number to describe the dive shown below: forward ______ somersaults

94. PRODUCT LABELING Several mixed numbers appear on the label shown below. Write each mixed number as an improper fraction.

95. READING METERS

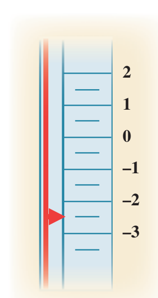
a. Use a mixed number to describe the value to which the arrow is currently pointing.

b. If the arrow moves twelve tick marks to the left, to what value will it be pointing?

96. READING METERS

a. Use a mixed number to describe the value to which the arrow is currently pointing.

b. If the arrow moves up six tick marks, to what value will it be pointing?



97. ONLINE SHOPPING A mother is ordering a pair of jeans for her daughter from the screen shown below. If the daughter's height is $60\frac{3}{4}$ in. and her waist is $24\frac{1}{2}$ in., on what size and what cut (regular or slim) should the mother point and click?

Girl's jeans- regular cut

Size	7	8	10	12	14	16
Height	50-52	52-54	54-56	$56\frac{1}{4}$-$58\frac{1}{2}$	59-61	61-62
Waist	$22\frac{1}{4}$-$22\frac{3}{4}$	$22\frac{3}{4}$-$23\frac{1}{4}$	$23\frac{3}{4}$-$24\frac{1}{4}$	$24\frac{3}{4}$-$25\frac{1}{4}$	$25\frac{3}{4}$-$26\frac{1}{4}$	$26\frac{1}{4}$-28

Girl's jeans- slim cut

Size	7	8	10	12	14	16
Height	50-52	52-54	54-56	$56\frac{1}{2}$-$58\frac{1}{2}$	59-61	61-62
Waist	$20\frac{3}{4}$-$21\frac{1}{4}$	$21\frac{1}{4}$-$21\frac{3}{4}$	$22\frac{1}{4}$-$22\frac{3}{4}$	$23\frac{1}{4}$-$23\frac{3}{4}$	$24\frac{1}{4}$-$24\frac{3}{4}$	25-$26\frac{1}{2}$

To order:
Point arrow to proper size/cut and click

98. SEWING Use the following table to determine the number of yards of fabric needed . . .

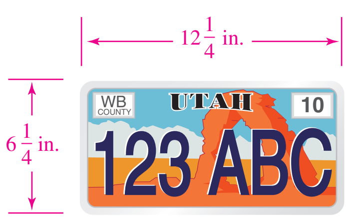
a. to make a size 16 top if the fabric to be used is 60 inches wide.

b. to make size 18 pants if the fabric to be used is 45 inches wide.

8767 Pattern
stitch'n save

SIZES	8	10	12	14	16	18	20	
Top								
45"	$2\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{3}{8}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	**Yds**
60"	2	2	$2\frac{1}{8}$	$2\frac{1}{8}$	$2\frac{1}{8}$	$2\frac{1}{8}$	$2\frac{1}{8}$	
Pants								
45"	$2\frac{5}{8}$	$2\frac{5}{8}$	$2\frac{5}{8}$	$2\frac{5}{8}$	$2\frac{5}{8}$	$2\frac{5}{8}$	$2\frac{5}{8}$	**Yds**
60"	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{4}$	$2\frac{1}{4}$	$2\frac{1}{4}$	$2\frac{1}{2}$	

99. LICENSE PLATES Find the area of the license plate shown below.

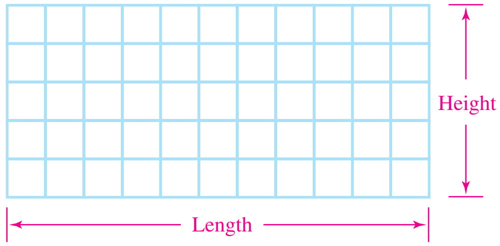


100. GRAPH PAPER Mathematicians use specially marked paper, called graph paper, when drawing figures. It is made up of squares that are $\frac{1}{4}$-inch long by $\frac{1}{4}$-inch high.

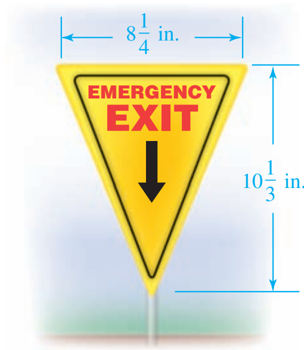
a. Find the length of the piece of graph paper shown below.

b. Find its height.

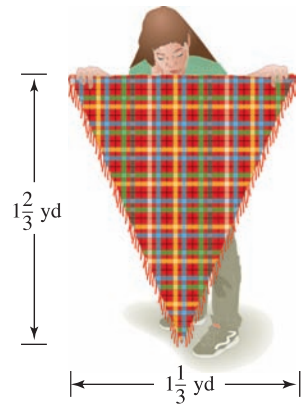
c. What is the area of the piece of graph paper?



101. EMERGENCY EXITS The following sign marks the emergency exit on a school bus. Find the area of the sign.



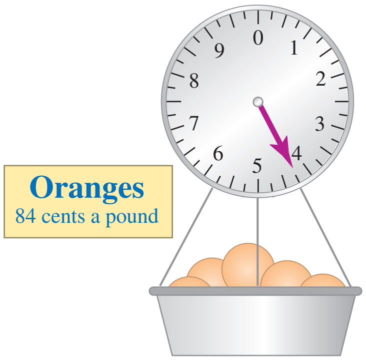
102. CLOTHING DESIGN Find the number of square yards of material needed to make the triangular-shaped shawl shown in the illustration.



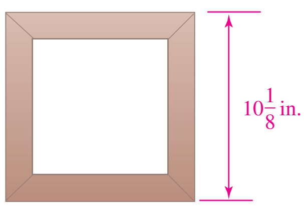
103. CALORIES A company advertises that its mints contain only $3\frac{1}{5}$ calories a piece. What is the calorie intake if you eat an entire package of 20 mints?

104. CEMENT MIXERS A cement mixer can carry $9\frac{1}{2}$ cubic yards of concrete. If it makes 8 trips to a job site, how much concrete will be delivered to the site?

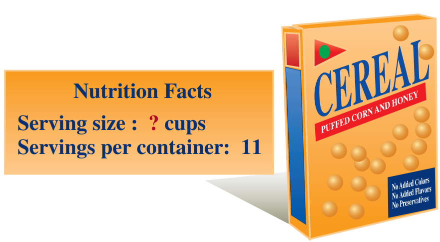
105. SHOPPING In the illustration, what is the cost of buying the fruit in the scale? Give your answer in cents and in dollars.



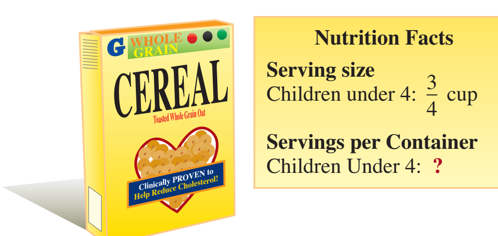
106. PICTURE FRAMES How many inches of molding is needed to make the square picture frame below?



107. BREAKFAST CEREAL A box of cereal contains about $13\frac{3}{4}$ cups. Refer to the nutrition label shown below and determine the recommended size of one serving.



108. BREAKFAST CEREAL A box of cereal contains about $14\frac{1}{4}$ cups. Refer to the nutrition label shown below. Determine how many servings there are for children under 4 in one box.



109. CATERING How many people can be served $\frac{1}{3}$-pound hamburgers if a caterer purchases 200 pounds of ground beef?

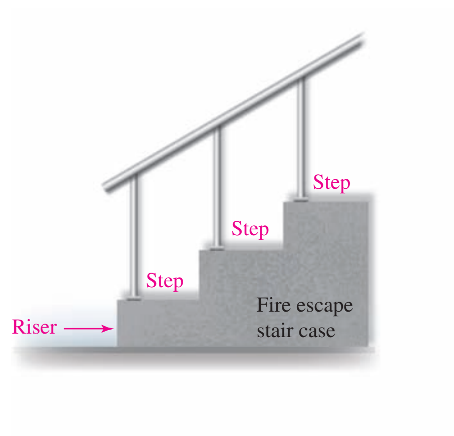
110. SUBDIVISIONS A developer donated to the county 100 of the 1,000 acres of land she owned. She divided the remaining acreage into $1\frac{1}{3}$-acre lots. How many lots were created?

111. HORSE RACING The race tracks on which thoroughbred horses run are marked off in $\frac{1}{8}$-mile-long segments called *furlongs*. How many furlongs are there in a $1\frac{1}{16}$-mile race?

112. FIRE ESCAPES Part of the fire escape stairway for one story of an office building is shown below. Each riser is $7\frac{1}{2}$ inches high and each story of the building is 105 inches high.

a. How many stairs are there in one story of the fire escape stairway?

b. If the building has 43 stories, how many stairs are there in the entire fire escape stairway?



WRITING

113. Explain the difference between $2\frac{3}{4}$ and $2\left(\frac{3}{4}\right)$.

114. Give three examples of how you use mixed numbers in daily life.

REVIEW

Find the LCM of the given numbers.

115. 5, 12, 15

116. 8, 12, 16

Find the GCF of the given numbers.

117. 12, 68, 92

118. 24, 36, 40

SECTION 3.6
Adding and Subtracting Mixed Numbers

Objectives
1. Add mixed numbers.
2. Add mixed numbers in vertical form.
3. Subtract mixed numbers.
4. Solve application problems by adding and subtracting mixed numbers.

In this section, we discuss several methods for adding and subtracting mixed numbers.

1 Add mixed numbers.

We can add mixed numbers by writing them as improper fractions. To do so, we follow these steps.

Adding Mixed Numbers: Method 1

1. Write each mixed number as an improper fraction.
2. Write each improper fraction as an equivalent fraction with a denominator that is the LCD.
3. Add the fractions.
4. Write the result as a mixed number, if desired.

Method 1 works well when the whole-number parts of the mixed numbers are small.

EXAMPLE 1 Add: $4\frac{1}{6} + 2\frac{3}{4}$

Strategy We will write each mixed number as an improper fraction, and then use the rule for adding two fractions that have different denominators.

WHY We cannot add the mixed numbers as they are; their fractional parts are not similar objects.

$$4\frac{1}{6} + 2\frac{3}{4}$$

Four and one-sixth Two and three-fourths

Solution

$$4\frac{1}{6} + 2\frac{3}{4} = \frac{25}{6} + \frac{11}{4}$$ Write $4\frac{1}{6}$ and $2\frac{3}{4}$ as improper fractions.

By inspection, we see that the lowest common denominator is 12.

$$= \frac{25 \cdot 2}{6 \cdot 2} + \frac{11 \cdot 3}{4 \cdot 3}$$ To build $\frac{25}{6}$ and $\frac{11}{4}$ so that their denominators are 12, multiply each by a form of 1.

$$= \frac{50}{12} + \frac{33}{12}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{83}{12}$$ Add the numerators and write the sum over the common denominator 12. The result is an improper fraction.

$$= 6\frac{11}{12}$$ Write the improper fraction $\frac{83}{12}$ as a mixed number.

$$\begin{array}{r} 6 \\ 12\overline{)83} \\ -72 \\ \hline 11 \end{array}$$

Self Check 1

Add: $3\frac{2}{3} + 1\frac{1}{5}$

Now Try **Problem 13**

Success Tip We can use rounding to check the results when adding (or subtracting) mixed numbers. To check the answer $6\frac{11}{12}$ from Example 1, we proceed as follows:

$$4\frac{1}{6} + 2\frac{3}{4} \approx 4 + 3 = 7$$

Since $\frac{1}{6}$ is less than $\frac{1}{2}$, round $4\frac{1}{6}$ down to 4.
Since $\frac{3}{4}$ is greater than $\frac{1}{2}$, round $2\frac{3}{4}$ up to 3.

Since $6\frac{11}{12}$ is close to 7, it is a reasonable answer.

Self Check 2

Add: $-4\frac{1}{12} + 2\frac{1}{4}$

Now Try **Problem 17**

EXAMPLE 2

Add: $-3\frac{1}{8} + 1\frac{1}{2}$

Strategy We will write each mixed number as an improper fraction, and then use the rule for adding two fractions that have different denominators.

WHY We cannot add the mixed numbers as they are; their fractional parts are not similar objects.

$$-3\frac{1}{8} + 1\frac{1}{2}$$

Negative three and one-eighth — One and one-half

Solution

$$-3\frac{1}{8} + 1\frac{1}{2} = -\frac{25}{8} + \frac{3}{2}$$ Write $-3\frac{1}{8}$ and $1\frac{1}{2}$ as improper fractions.

Since the smallest number the denominators 8 and 2 divide exactly is 8, the LCD is 8. We will only need to build an equivalent fraction for $\frac{3}{2}$.

$$= -\frac{25}{8} + \frac{3}{2} \cdot \frac{4}{4}$$ To build $\frac{3}{2}$ so that its denominator is 8, multiply it by a form of 1.

$$= -\frac{25}{8} + \frac{12}{8}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{-25 + 12}{8}$$ Add the numerators and write the sum over the common denominator 8.

$$= \frac{-13}{8}$$ Use the rule for adding integers that have different signs: $-25 + 12 = -13$.

$$= -1\frac{5}{8}$$ Write $\frac{-13}{8}$ as a negative mixed number by dividing 13 by 8.

We can also add mixed numbers by adding their whole-number parts and their fractional parts. To do so, we follow these steps.

Adding Mixed Numbers: Method 2

1. Write each mixed number as the sum of a whole number and a fraction.
2. Use the commutative property of addition to write the whole numbers together and the fractions together.
3. Add the whole numbers and the fractions separately.
4. Write the result as a mixed number, if necessary.

Method 2 works well when the whole number parts of the mixed numbers are large.

EXAMPLE 3 Add: $168\frac{3}{7} + 85\frac{2}{9}$

Strategy We will write each mixed number as the sum of a whole number and a fraction. Then we will add the whole numbers and the fractions separately.

WHY If we change each mixed number to an improper fraction, build equivalent fractions, and add, the resulting numerators will be very large and difficult to work with.

Solution

We will write the solution in *horizontal* form.

$$168\frac{3}{7} + 85\frac{2}{9} = 168 + \frac{3}{7} + 85 + \frac{2}{9}$$ Write each mixed number as the sum of a whole number and a fraction.

$$= 168 + 85 + \frac{3}{7} + \frac{2}{9}$$ Use the commutative property of addition to change the order of the addition so that the whole numbers are together and the fractions are together.

$$= 253 + \frac{3}{7} + \frac{2}{9}$$ Add the whole numbers.

$$\begin{array}{r} {}^{1\,1} \\ 168 \\ +\ 85 \\ \hline 253 \end{array}$$

$$= 253 + \frac{3}{7}\cdot\frac{9}{9} + \frac{2}{9}\cdot\frac{7}{7}$$ Prepare to add the fractions. To build $\frac{3}{7}$ and $\frac{2}{9}$ so that their denominators are 63, multipy each by a form of 1.

$$= 253 + \frac{27}{63} + \frac{14}{63}$$ Multiply the numerators. Multiply the denominators.

$$= 253 + \frac{41}{63}$$ Add the numerators and write the sum over the common denominator 63.

$$\begin{array}{r} {}^{1} \\ 27 \\ +\ 14 \\ \hline 41 \end{array}$$

$$= 253\frac{41}{63}$$ Write the sum as a mixed number.

Caution! If we use method 1 to add the mixed numbers in Example 3, the numbers we encounter are very large. As expected, the result is the same: $253\frac{41}{63}$.

$$168\frac{3}{7} + 85\frac{2}{9} = \frac{1{,}179}{7} + \frac{767}{9}$$ Write $168\frac{3}{7}$ and $85\frac{2}{9}$ as improper fractions.

$$= \frac{1{,}179}{7}\cdot\frac{9}{9} + \frac{767}{9}\cdot\frac{7}{7}$$ The LCD is 63.

$$= \frac{10{,}611}{63} + \frac{5{,}369}{63}$$ Note how large the numerators are.

$$= \frac{15{,}980}{63}$$ Add the numerators and write the sum over the common denominator 63.

$$= 253\frac{41}{63}$$ To write the improper fraction as a mixed number, divide 15,980 by 63.

Generally speaking, the larger the whole-number parts of the mixed numbers, the more difficult it becomes to add those mixed numbers using method 1.

2 Add mixed numbers in vertical form.

We can add mixed numbers quickly when they are written in **vertical form** by working in columns. The strategy is the same as in Example 2: Add whole numbers to whole numbers and fractions to fractions.

Self Check 3

Add: $275\frac{1}{6} + 81\frac{3}{5}$

Now Try Problem 21

Self Check 4

Add: $71\frac{5}{8} + 23\frac{1}{3}$

Now Try **Problem 25**

EXAMPLE 4

Add: $25\frac{3}{4} + 31\frac{1}{5}$

Strategy We will perform the addition in *vertical form* with the fractions in a column and the whole numbers lined up in columns. Then we will add the fractional parts and the whole-number parts separately.

WHY It is often easier to add the fractional parts and the whole-number parts of mixed numbers vertically—especially if the whole-number parts contain two or more digits, such as 25 and 31.

Solution

Write the mixed numbers in vertical form.
Build $\frac{3}{4}$ and $\frac{1}{5}$ so that their denominators are 20.
Add the fractions separately.
Add the whole numbers separately.

$$\begin{array}{r} 25\frac{3}{4} \\ +\ 31\frac{1}{5} \\ \hline \end{array} = \begin{array}{r} 25\frac{3}{4}\cdot\frac{5}{5} \\ +\ 31\frac{1}{5}\cdot\frac{4}{4} \\ \hline \end{array} = \begin{array}{r} 25\frac{15}{20} \\ +\ 31\frac{4}{20} \\ \hline \frac{19}{20} \end{array} = \begin{array}{r} 25\frac{15}{20} \\ +\ 31\frac{4}{20} \\ \hline 56\frac{19}{20} \end{array}$$

The sum is $56\frac{19}{20}$.

Self Check 5

Add and simplify, if possible:
$68\frac{1}{6} + 37\frac{5}{18} + 52\frac{1}{9}$

Now Try **Problem 29**

EXAMPLE 5

Add and simplify, if possible: $75\frac{1}{12} + 43\frac{1}{4} + 54\frac{1}{6}$

Strategy We will write the problem in *vertical form*. We will make sure that the fractional part of the answer is in simplest form.

WHY When adding, subtracting, multiplying, or dividing fractions or mixed numbers, the answer should always be written in simplest form.

Solution

The LCD for $\frac{1}{12}, \frac{1}{4}$, and $\frac{1}{6}$ is 12.

Write the mixed numbers in vertical form.
Build $\frac{1}{4}$ and $\frac{1}{6}$ so that their denominators are 12.
Add the fractions separately.
Add the whole numbers separately.

$$\begin{array}{r} 75\frac{1}{12} \\ 43\frac{1}{4} \\ +\ 54\frac{1}{6} \\ \hline \end{array} = \begin{array}{r} 75\frac{1}{12} \\ 43\frac{1}{4}\cdot\frac{3}{3} \\ +\ 54\frac{1}{6}\cdot\frac{2}{2} \\ \hline \end{array} = \begin{array}{r} 75\frac{1}{12} \\ 43\frac{3}{12} \\ +\ 54\frac{2}{12} \\ \hline \frac{6}{12} \end{array} = \begin{array}{r} \overset{1\,1}{75}\frac{1}{12} \\ 43\frac{3}{12} \\ +\ 54\frac{2}{12} \\ \hline 172\frac{6}{12} \end{array} = 172\frac{1}{2}$$

Simplify: $\frac{6}{12} = \frac{\overset{1}{\cancel{6}}}{2\cdot\underset{1}{\cancel{6}}} = \frac{1}{2}$.

The sum is $172\frac{1}{2}$.

When we add mixed numbers, sometimes the sum of the fractions is an improper fraction.

EXAMPLE 6 Add: $45\frac{2}{3} + 96\frac{4}{5}$

Strategy We will write the problem in *vertical form*. We will make sure that the fractional part of the answer is in simplest form.

WHY When adding, subtracting, multiplying, or dividing fractions or mixed numbers, the answer should always be written in simplest form.

Solution

The LCD for $\frac{2}{3}$ and $\frac{4}{5}$ is 15.

Write the mixed numbers in vertical form.
Build $\frac{2}{3}$ and $\frac{4}{5}$ so that their denominators are 15.
Add the fractions separately.
Add the whole numbers separately.

$$\begin{array}{r} 45\frac{2}{3} \\ +\ 96\frac{4}{5} \\ \hline \end{array} = \begin{array}{r} 45\frac{2}{3}\cdot\frac{5}{5} \\ +\ 96\frac{4}{5}\cdot\frac{3}{3} \\ \hline \end{array} = \begin{array}{r} 45\frac{10}{15} \\ +\ 96\frac{12}{15} \\ \hline \frac{22}{15} \end{array} = \begin{array}{r} 45\frac{10}{15} \\ +\ 96\frac{12}{15} \\ \hline 141\frac{22}{15} \end{array}$$

The fractional part of the answer is greater than 1.

Since we don't want an improper fraction in the answer, we write $\frac{22}{15}$ as a mixed number. Then we *carry* 1 from the fraction column to the whole-number column.

$$141\frac{22}{15} = 141 + \frac{22}{15}$$ Write the mixed number as the sum of a whole number and a fraction.

$$= 141 + 1\frac{7}{15}$$ To write the improper fraction as a mixed number divide 22 by 15.

$$= 142\frac{7}{15}$$ Carry the 1 and add it to 141 to get 142.

$$\begin{array}{r} 1 \\ 15\overline{)22} \\ -\ 15 \\ \hline 7 \end{array}$$

Self Check 6

Add: $76\frac{11}{12} + 49\frac{5}{8}$

Now Try **Problem 33**

3 Subtract mixed numbers.

Subtracting mixed numbers is similar to adding mixed numbers.

EXAMPLE 7 Subtract and simplify, if possible: $16\frac{7}{10} - 9\frac{8}{15}$

Strategy We will perform the subtraction in *vertical form* with the fractions in a column and the whole numbers lined up in columns. Then we will subtract the fractional parts and the whole-number parts separately.

WHY It is often easier to subtract the fractional parts and the whole-number parts of mixed numbers vertically.

Self Check 7

Subtract and simplify, if possible:

$12\frac{9}{20} - 8\frac{1}{30}$

Now Try **Problem 37**

Solution

The LCD for $\frac{7}{10}$ and $\frac{8}{15}$ is 30.

Write the mixed numbers in vertical form.

Build $\frac{7}{10}$ and $\frac{8}{15}$ so that their denominators are 30.

Subtract the fractions separately.

Subtract the whole numbers separately.

$$\begin{array}{r} 16\frac{7}{10} \\ -\;9\frac{8}{15} \\ \hline \end{array} = \begin{array}{r} 16\frac{7}{10}\cdot\frac{3}{3} \\ -\;9\frac{8}{15}\cdot\frac{2}{2} \\ \hline \end{array} = \begin{array}{r} 16\frac{21}{30} \\ -\;9\frac{16}{30} \\ \hline \frac{5}{30} \end{array} = \begin{array}{r} 16\frac{21}{30} \\ -\;9\frac{16}{30} \\ \hline 7\frac{5}{30} \end{array} = 7\frac{1}{6}$$

Simplify:

$\frac{5}{30} = \frac{\cancel{5}^{1}}{\cancel{5}_{1} \cdot 6} = \frac{1}{6}$.

The difference is $7\frac{1}{6}$.

Subtraction of mixed numbers (like subtraction of whole numbers) sometimes involves borrowing. When the fraction we are subtracting is greater than the fraction we are subtracting it from, it is necessary to borrow.

Self Check 8

Subtract: $258\frac{3}{4} - 175\frac{15}{16}$

Now Try **Problem 41**

EXAMPLE 8

Subtract: $34\frac{1}{8} - 11\frac{2}{3}$

Strategy We will perform the subtraction in *vertical form* with the fractions in a column and the whole numbers lined up in columns. Then we will subtract the fractional parts and the whole-number parts separately.

WHY It is often easier to subtract the fractional parts and the whole-number parts of mixed numbers vertically.

Solution

The LCD for $\frac{1}{8}$ and $\frac{2}{3}$ is 24.

Write the mixed number in vertical form.

Build $\frac{1}{8}$ and $\frac{2}{3}$ so that their denominators are 24.

$$\begin{array}{r} 34\frac{1}{8} \\ -\;11\frac{2}{3} \\ \hline \end{array} = \begin{array}{r} 34\frac{1}{8}\cdot\frac{3}{3} \\ -\;11\frac{2}{3}\cdot\frac{8}{8} \\ \hline \end{array} = \begin{array}{r} 34\frac{3}{24} \\ -\;11\frac{16}{24} \\ \hline \end{array}$$

Note that $\frac{16}{24}$ is greater than $\frac{3}{24}$.

Since $\frac{16}{24}$ is greater than $\frac{3}{24}$, borrow 1 (in the form of $\frac{24}{24}$) from 34 and add it to $\frac{3}{24}$ to get $\frac{27}{24}$.

Subtract the fractions separately.

Subtract the whole numbers separately.

$$\begin{array}{r} \overset{3}{\cancel{34}}\,\overset{3}{}\frac{3}{24} + \frac{24}{24} \\ -\;11\frac{16}{24} \\ \hline \end{array} = \begin{array}{r} 33\frac{27}{24} \\ -\;11\frac{16}{24} \\ \hline \frac{11}{24} \end{array} = \begin{array}{r} 33\frac{27}{24} \\ -\;11\frac{16}{24} \\ \hline 22\frac{11}{24} \end{array}$$

The difference is $22\frac{11}{24}$.

Success Tip We can use rounding to check the results when subtracting mixed numbers. To check the answer $22\frac{11}{24}$ from Example 8, we proceed as follows:

$$34\frac{1}{8} - 11\frac{2}{3} \approx 34 - 12 = 22$$

Since $\frac{1}{8}$ is less than $\frac{1}{2}$, round $34\frac{1}{8}$ down to 34.
Since $\frac{2}{3}$ is greater than $\frac{1}{2}$, round $11\frac{2}{3}$ up to 12.

Since $22\frac{11}{24}$ is close to 22, it is a reasonable answer.

EXAMPLE 9 Subtract: $419 - 53\frac{11}{16}$

Strategy We will write the numbers in vertical form and borrow 1 $\left(\text{in the form of } \frac{16}{16}\right)$ from 419.

WHY In the fraction column, we need to have a fraction from which to subtract $\frac{11}{16}$.

Solution

Write the mixed number in vertical form.
Borrow 1 (in the form of $\frac{16}{16}$) from 419. Then subtract the fractions separately.
Subtract the whole numbers separately. This also requires borrowing.

$$\begin{array}{r} 419 \\ -\ 53\frac{11}{16} \\ \hline \end{array} = \begin{array}{r} 418\frac{16}{16} \\ -\ 53\frac{11}{16} \\ \hline 365\frac{5}{16} \end{array} = \begin{array}{r} \overset{3\,11}{\cancel{4}\cancel{1}8}\frac{16}{16} \\ -\ 53\frac{11}{16} \\ \hline 365\frac{5}{16} \end{array}$$

The difference is $365\frac{5}{16}$.

Self Check 9

Subtract: $2{,}300 - 129\frac{31}{32}$

Now Try **Problem 45**

4 Solve application problems by adding and subtracting mixed numbers.

EXAMPLE 10 ***Horse Racing*** In order to become the *Triple Crown Champion,* a thoroughbred horse must win three races: the Kentucky Derby ($1\frac{1}{4}$ miles long), the Preakness Stakes ($1\frac{3}{16}$ miles long), and the Belmont Stakes ($1\frac{1}{2}$ miles long). What is the combined length of the three races of the Triple Crown?


Focus on Sport/Getty Images

Affirmed, in 1978, was the last of only 11 horses in history to win the Triple Crown.

Analyze

- The Kentucky Derby is $1\frac{1}{4}$ miles long.
- The Preakness Stakes is $1\frac{3}{16}$ miles long.
- The Belmont Stakes is $1\frac{1}{2}$ miles long.
- What is the combined length of the three races?

Self Check 10

SALADS A three-bean salad calls for one can of green beans ($14\frac{1}{2}$ ounces), one can of garbanzo beans ($10\frac{3}{4}$ ounces), and one can of kidney beans ($15\frac{7}{8}$ ounces). How many ounces of beans are called for in the recipe?

Now Try **Problem 89**

Form The key phrase *combined length* indicates addition.

We translate the words of the problem to numbers and symbols.

The combined length of the three races	is equal to	the length of the Kentucky Derby	plus	the length of the Preakness Stakes	plus	the length of the Belmont Stakes.
The combined length of the three races	$=$	$1\frac{1}{4}$	$+$	$1\frac{3}{16}$	$+$	$1\frac{1}{2}$

Solve To find the sum, we will write the mixed numbers in vertical form. To add in the fraction column, the LCD for $\frac{1}{4}$, $\frac{3}{16}$, and $\frac{1}{2}$ is 16.

Build $\frac{1}{4}$ and $\frac{1}{2}$ so that their denominators are 16.

Add the fractions separately.

Add the whole numbers separately.

$$
\begin{array}{rcrcrcr}
1\frac{1}{4} & = & 1\frac{1}{4}\cdot\frac{4}{4} & = & 1\frac{4}{16} & = & 1\frac{4}{16} \\[6pt]
1\frac{3}{16} & = & 1\frac{3}{16} & = & 1\frac{3}{16} & = & 1\frac{3}{16} \\[6pt]
+\,1\frac{1}{2} & = & +\,1\frac{1}{2}\cdot\frac{8}{8} & = & +\,1\frac{8}{16} & = & +\,1\frac{8}{16} \\
\hline
 & & & & \frac{15}{16} & & 3\frac{15}{16}
\end{array}
$$

State The combined length of the three races of the Triple Crown is $3\frac{15}{16}$ miles.

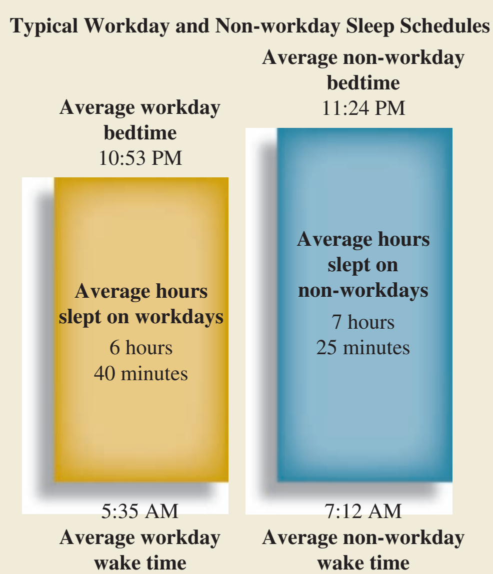
Check We can estimate to check the result. If we round $1\frac{1}{4}$ down to 1, round $1\frac{3}{16}$ down to 1, and round $1\frac{1}{2}$ up to 2, the approximate combined length of the three races is $1 + 1 + 2 = 4$ miles. Since $3\frac{15}{16}$ is close to 4, the result seems reasonable.

THINK IT THROUGH

"Americans are not getting the sleep they need which may affect their ability to perform well during the workday."

National Sleep Foundation Report, 2008

The 1,000 people who took part in the 2008 *Sleep in America* poll were asked when they typically wake up, when they go to bed, and how long they sleep on both workdays and non-workdays. The results are shown on the right. Write the average hours slept on a workday and on a non-workday as mixed numbers. How much longer does the average person sleep on a non-workday?



(Source: National Sleep Foundation, 2008)

EXAMPLE 11 ***Baking*** How much butter is left in a 10-pound tub if $2\frac{2}{3}$ pounds are used for a wedding cake?

Analyze

- The tub contained 10 pounds of butter.
- $2\frac{2}{3}$ pounds of butter are used for a cake.
- How much butter is left in the tub?

Form The key phrase *how much butter is left* indicates subtraction.

We translate the words of the problem to numbers and symbols.

The amount of butter left in the tub	is equal to	the amount of butter in one tub	minus	the amount of butter used for the cake.
The amount of butter left in the tub	=	10	−	$2\frac{2}{3}$

Solve To find the difference, we will write the numbers in vertical form and borrow 1 (in the form of $\frac{3}{3}$) from 10.

In the fraction column, we need to have a fraction from which to subtract $\frac{2}{3}$.

Subtract the fractions separately.

Subtract the whole numbers separately.

$$\begin{array}{r} 10 \\ -\ 2\frac{2}{3} \\ \hline \end{array} \quad = \quad \begin{array}{r} \overset{9}{\cancel{10}}\frac{3}{3} \\ -\ 2\frac{2}{3} \\ \hline \frac{1}{3} \end{array} \quad = \quad \begin{array}{r} \overset{9}{\cancel{10}}\frac{3}{3} \\ -\ 2\frac{2}{3} \\ \hline 7\frac{1}{3} \end{array}$$

State There are $7\frac{1}{3}$ pounds of butter left in the tub.

Check We can check using addition. If $2\frac{2}{3}$ pounds of butter were used and $7\frac{1}{3}$ pounds of butter are left in the tub, then the tub originally contained $2\frac{2}{3} + 7\frac{1}{3} = 9\frac{3}{3} = 10$ pounds of butter. The result checks.

Self Check 11

TRUCKING The mixing barrel of a cement truck holds 9 cubic yards of concrete. How much concrete is left in the barrel if $6\frac{3}{4}$ cubic yards have already been unloaded?

Now Try **Problem 95**

ANSWER TO SELF CHECKS

1. $4\frac{13}{15}$ **2.** $-1\frac{5}{6}$ **3.** $356\frac{23}{30}$ **4.** $94\frac{23}{24}$ **5.** $157\frac{5}{9}$ **6.** $126\frac{13}{24}$ **7.** $4\frac{5}{12}$ **8.** $82\frac{13}{16}$ **9.** $2{,}170\frac{1}{32}$ **10.** $41\frac{1}{8}$ oz **11.** $2\frac{1}{4}$ yd^3

SECTION 3.6 STUDY SET

VOCABULARY

Fill in the blanks.

1. A _______ number, such as $1\frac{7}{8}$, contains a whole-number part and a fractional part.

2. We can add (or subtract) mixed numbers quickly when they are written in _______ form by working in columns.

3. To add (or subtract) mixed numbers written in vertical form, we add (or subtract) the _________ separately and the _______ numbers separately.

4. Fractions such as $\frac{11}{8}$, that are greater than or equal to 1, are called _________ fractions.

5. Consider the following problem:

$$\begin{array}{r} 36\frac{5}{7} \\ +\ 42\frac{4}{7} \\ \hline 78\frac{9}{7} = 78 + 1\frac{2}{7} = 79\frac{2}{7} \end{array}$$

Since we don't want an improper fraction in the answer, we write $\frac{9}{7}$ as $1\frac{2}{7}$, ______ the 1, and add it to 78 to get 79.

6. Consider the following problem:

$$\begin{array}{rcl} 86\frac{1}{3} & = & 8\overset{5}{\cancel{6}}\frac{1}{3} + \frac{3}{3} \\ -\ 24\frac{2}{3} & = & -\ 24\frac{2}{3} \\ \hline \end{array}$$

To subtract in the fraction column, we ________ 1 from 86 in the form of $\frac{3}{3}$.

CONCEPTS

7. a. For $76\frac{3}{4}$, list the whole-number part and the fractional part.

b. Write $76\frac{3}{4}$ as a sum.

8. Use the commutative property of addition to rewrite the following expression with the whole numbers together and the fractions together. *You do not have to find the answer.*

$$14 + \frac{5}{8} + 53 + \frac{1}{6}$$

9. The *denominators* of two fractions are given. Find the least common denominator.

a. 3 and 4 **b.** 5 and 6

c. 6 and 9 **d.** 8 and 12

10. Simplify.

a. $9\frac{17}{16}$ **b.** $1{,}288\frac{7}{3}$

c. $16\frac{12}{8}$ **d.** $45\frac{24}{20}$

NOTATION

Fill in the blanks to complete each solution.

11.
$$\begin{array}{rcrcr} 6\frac{3}{5} & = & 6\frac{3}{5}\cdot\frac{7}{7} & = & 6\frac{\square}{35} \\ +\ 3\frac{2}{7} & = & +\ 3\frac{2}{7}\cdot\frac{\square}{\square} & = & +\ 3\frac{10}{\square} \\ \hline & & & & 9\frac{\square}{\square} \end{array}$$

12.
$$\begin{array}{rcrcrcrcr} 67\frac{3}{8} & = & 67\frac{3}{8}\cdot\frac{\square}{\square} & = & 67\frac{9}{24} & = & \overset{6}{6\cancel{7}}\frac{9}{24} + \frac{\square}{\square} & = & 66\frac{\square}{24} \\ -\ 23\frac{2}{3} & = & -\ 23\frac{2}{3}\cdot\frac{8}{8} & = & -\ 23\frac{\square}{24} & = & -\ 23\frac{16}{24} & = & -\ 23\frac{16}{24} \\ \hline & & & & & & & & \square\frac{\square}{24} \end{array}$$

GUIDED PRACTICE

Add. See Example 1.

13. $1\frac{1}{4} + 2\frac{1}{3}$

14. $2\frac{2}{5} + 3\frac{1}{4}$

15. $2\frac{1}{3} + 4\frac{2}{5}$

16. $4\frac{1}{3} + 1\frac{1}{7}$

Add. See Example 2.

17. $-4\frac{1}{8} + 1\frac{3}{4}$

18. $-3\frac{11}{15} + 2\frac{1}{5}$

19. $-6\frac{5}{6} + 3\frac{2}{3}$

20. $-6\frac{3}{14} + 1\frac{2}{7}$

Add. See Example 3.

21. $334\frac{1}{7} + 42\frac{2}{3}$

22. $259\frac{3}{8} + 40\frac{1}{3}$

23. $667\frac{1}{5} + 47\frac{3}{4}$

24. $568\frac{1}{6} + 52\frac{3}{4}$

Add. See Example 4.

25. $41\frac{2}{9} + 18\frac{2}{5}$

26. $60\frac{3}{11} + 24\frac{2}{3}$

27. $89\frac{6}{11} + 43\frac{1}{3}$

28. $77\frac{5}{8} + 55\frac{1}{7}$

Add and simplify, if possible. See Example 5.

29. $14\frac{1}{4} + 29\frac{1}{20} + 78\frac{3}{5}$

30. $11\frac{1}{12} + 59\frac{1}{4} + 82\frac{1}{6}$

31. $106\frac{5}{18} + 22\frac{1}{2} + 19\frac{1}{9}$

32. $75\frac{2}{5} + 43\frac{7}{30} + 54\frac{1}{3}$

Add and simplify, if possible. See Example 6.

33. $39\frac{5}{8} + 62\frac{11}{12}$

34. $53\frac{5}{6} + 47\frac{3}{8}$

35. $82\frac{8}{9} + 46\frac{11}{15}$

36. $44\frac{2}{9} + 76\frac{20}{21}$

Subtract and simplify, if possible. See Example 7.

37. $19\frac{11}{12} - 9\frac{2}{3}$

38. $32\frac{2}{3} - 7\frac{1}{6}$

39. $21\frac{5}{6} - 8\frac{3}{10}$

40. $41\frac{2}{5} - 6\frac{3}{20}$

Subtract. See Example 8.

41. $47\frac{1}{11} - 15\frac{2}{3}$

42. $58\frac{4}{11} - 15\frac{1}{2}$

43. $84\frac{5}{8} - 12\frac{6}{7}$

44. $95\frac{4}{7} - 23\frac{5}{6}$

Subtract. See Example 9.

45. $674 - 94\frac{11}{15}$

46. $437 - 63\frac{6}{23}$

47. $112 - 49\frac{9}{32}$

48. $221 - 88\frac{35}{64}$

TRY IT YOURSELF

Add or subtract and simplify, if possible.

49. $140\frac{5}{6} - 129\frac{4}{5}$

50. $291\frac{1}{4} - 289\frac{1}{12}$

51. $4\frac{1}{6} + 1\frac{1}{5}$

52. $2\frac{2}{5} + 3\frac{1}{4}$

53. $5\frac{1}{2} + 3\frac{4}{5}$

54. $6\frac{1}{2} + 2\frac{2}{3}$

55. $2 + 1\frac{7}{8}$

56. $3\frac{3}{4} + 5$

57. $8\frac{7}{9} - 3\frac{1}{9}$

58. $9\frac{9}{10} - 6\frac{3}{10}$

59. $140\frac{3}{16} - 129\frac{3}{4}$

60. $442\frac{1}{8} - 429\frac{2}{3}$

61. $380\frac{1}{6} + 17\frac{1}{4}$

62. $103\frac{1}{2} + 210\frac{2}{5}$

63. $-2\frac{5}{6} + 1\frac{3}{8}$

64. $-4\frac{5}{9} + 2\frac{1}{6}$

65. $3\frac{1}{4} + 4\frac{1}{4}$

66. $2\frac{1}{8} + 3\frac{3}{8}$

67. $-3\frac{3}{4} + \left(-1\frac{1}{2}\right)$

68. $-3\frac{2}{3} + \left(-1\frac{4}{5}\right)$

69. $7 - \frac{2}{3}$

70. $6 - \frac{1}{8}$

71. $12\frac{1}{2} + 5\frac{3}{4} + 35\frac{1}{6}$

72. $31\frac{1}{3} + 20\frac{2}{5} + 10\frac{1}{15}$

73. $16\frac{1}{4} - 13\frac{3}{4}$

74. $40\frac{1}{7} - 19\frac{6}{7}$

75. $-4\frac{5}{8} - 1\frac{1}{4}$

76. $-2\frac{1}{16} - 3\frac{7}{8}$

77. $6\frac{5}{8} - 3$

78. $10\frac{1}{2} - 6$

79. $\frac{7}{3} + 2$

80. $\frac{9}{7} + 3$

81. $58\frac{7}{8} + 340\frac{1}{2} + 61\frac{3}{4}$

82. $191\frac{1}{2} + 233\frac{1}{16} + 16\frac{5}{8}$

83. $9 - 8\frac{3}{4}$

84. $11 - 10\frac{4}{5}$

APPLICATIONS

85. AIR TRAVEL A businesswoman's flight left Los Angeles and in $3\frac{3}{4}$ hours she landed in Minneapolis. She then boarded a commuter plane in Minneapolis and arrived at her final destination in $1\frac{1}{2}$ hours. Find the total time she spent on the flights.

86. SHIPPING A passenger ship and a cargo ship left San Diego harbor at midnight. During the first hour, the passenger ship traveled south at $16\frac{1}{2}$ miles per hour, while the cargo ship traveled north at a rate of $5\frac{1}{5}$ miles per hour. How far apart were they at 1:00 A.M.?

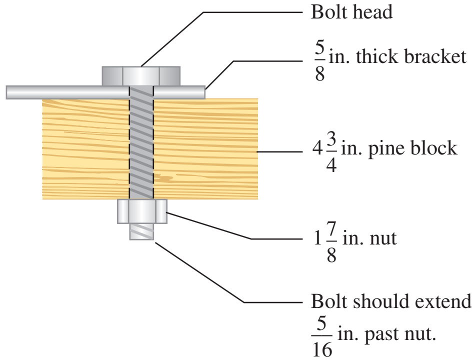
87. TRAIL MIX How many cups of trail mix will the recipe shown below make?

Trail Mix

A healthy snack–great for camping trips

$2\frac{3}{4}$ cups peanuts	$\frac{1}{3}$ cup coconut
$\frac{1}{2}$ cup sunflower seeds	$2\frac{2}{3}$ cups oat flakes
$\frac{2}{3}$ cup raisins	$\frac{1}{4}$ cup pretzels

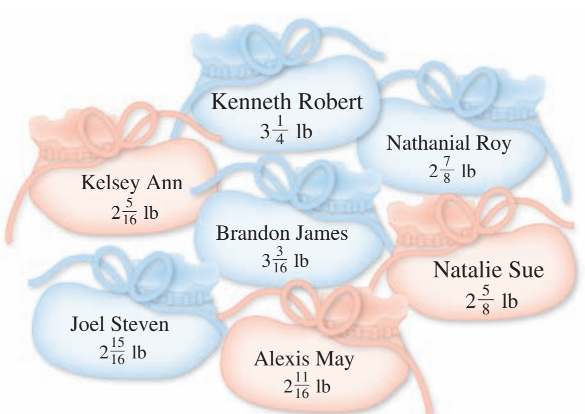
88. HARDWARE Refer to the illustration below. How long should the threaded part of the bolt be?



89. OCTUPLETS On January 26, 2009, at Kaiser Permanente Bellflower Medical Center in California, Nadya Suleman gave birth to eight babies. (The United States' first live octuplets were born in Houston in 1998 to Nkem Chukwu and Iyke Louis Udobi). Find the combined birthweights of the babies from the information shown below. (Source: The Nadya Suleman family website)

No. 1: Noah, male, $2\frac{11}{16}$ pounds
No. 2: Maliah, female, $2\frac{3}{4}$ pounds
No. 3: Isaiah, male, $3\frac{1}{4}$ pounds
No. 4: Nariah, female, $2\frac{1}{2}$ pounds
No. 5: Makai, male, $1\frac{1}{2}$ pounds
No. 6: Josiah, male, $2\frac{3}{4}$ pounds
No. 7: Jeremiah, male, $1\frac{15}{16}$ pounds
No. 8: Jonah, male, $2\frac{11}{16}$ pounds

90. SEPTUPLETS On November 19, 1997, at Iowa Methodist Medical Center, Bobbie McCaughey gave birth to seven babies. Find the combined birthweights of the babies from the following information. (Source: *Los Angeles Times*, Nov. 20, 1997)



91. HISTORICAL DOCUMENTS The Declaration of Independence on display at the National Archives in Washington, D.C., is $24\frac{1}{2}$ inches wide by $29\frac{3}{4}$ inches high. How many inches of molding would be needed to frame it?

92. STAMP COLLECTING The Pony Express Stamp, shown below, was issued in 1940. It is a favorite of collectors all over the world. A Postal Service document describes its size in an unusual way:

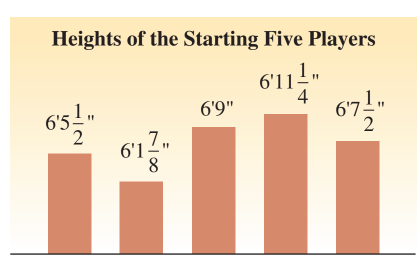
"The dimensions of the stamp are $\frac{84}{100}$ by $1\frac{44}{100}$ inches, arranged horizontally."

To display the stamp, a collector wants to frame it with gold braid. How many inches of braid are needed?

Smithsonian National Postal Museum

93. FREEWAY SIGNS A freeway exit sign is shown. How far apart are the Citrus Ave. and Grand Ave. exits?

94. BASKETBALL See the graph below. What is the difference in height between the tallest and the shortest of the starting players?



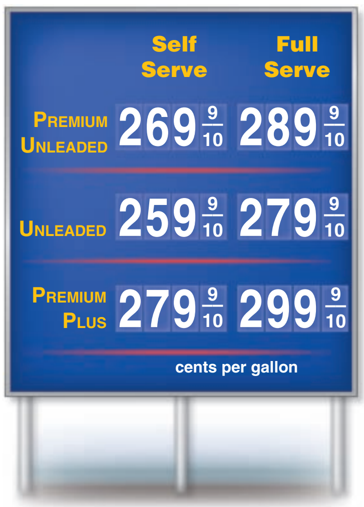
95. HOSE REPAIRS To repair a bad connector, a gardener removes $1\frac{1}{2}$ feet from the end of a 50-foot hose. How long is the hose after the repair?

96. HAIRCUTS A mother makes her child get a haircut when his hair measures 3 inches in length. His barber uses clippers with attachment #2 that leaves $\frac{3}{8}$-inch of hair. How many inches does the child's hair grow between haircuts?

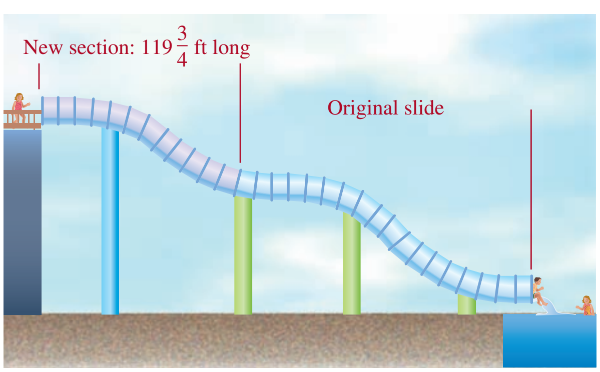
97. SERVICE STATIONS Use the service station sign below to answer the following questions.

a. What is the difference in price between the least and most expensive types of gasoline at the self-service pump?

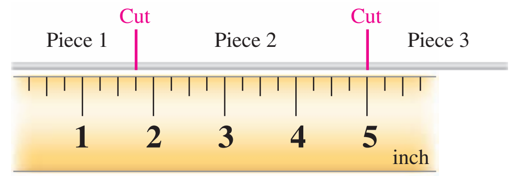
b. For each type of gasoline, how much more is the cost per gallon for full service compared to self service?



98. WATER SLIDES An amusement park added a new section to a water slide to create a slide $311\frac{5}{12}$ feet long. How long was the slide before the addition?



99. JEWELRY A jeweler cut a 7-inch-long silver wire into three pieces. To do this, he aligned a 6-inch-long ruler directly below the wire and made the proper cuts. Find the length of piece 2 of the wire.



100. SEWING To make some draperies, an interior decorator needs $12\frac{1}{4}$ yards of material for the den and $8\frac{1}{2}$ yards for the living room. If the material comes only in 21-yard bolts, how much will be left over after completing both sets of draperies?

WRITING

101. Of the methods studied to add mixed numbers, which do you like better, and why?

102. LEAP YEAR It actually takes Earth $365\frac{1}{4}$ days, give or take a few minutes, to make one revolution around the sun. Explain why every four years we add a day to the calendar to account for this fact.

103. Explain the process of simplifying $12\frac{7}{5}$.

104. Consider the following problem:

$$\begin{array}{r} 108\frac{1}{3} \\ -\ 99\frac{2}{3} \\ \hline \end{array}$$

a. Explain why borrowing is necessary.

b. Explain how the borrowing is done.

REVIEW

Perform each operation and simplify, if possible.

105. **a.** $3\dfrac{1}{2} + 1\dfrac{1}{4}$ **b.** $3\dfrac{1}{2} - 1\dfrac{1}{4}$

c. $3\dfrac{1}{2} \cdot 1\dfrac{1}{4}$ **d.** $3\dfrac{1}{2} \div 1\dfrac{1}{4}$

106. **a.** $5\dfrac{1}{10} + \dfrac{4}{5}$ **b.** $5\dfrac{1}{10} - \dfrac{4}{5}$

c. $5\dfrac{1}{10} \cdot \dfrac{4}{5}$ **d.** $5\dfrac{1}{10} \div \dfrac{4}{5}$

Objectives

1. Use the order of operations rule.
2. Solve application problems by using the order of operations rule.
3. Evaluate formulas.
4. Simplify complex fractions.

SECTION 3.7
Order of Operations and Complex Fractions

We have seen that the order of operations rule is used to evaluate expressions that contain more than one operation. In Chapter 1, we used it to evaluate expressions involving whole numbers, and in Chapter 2, we used it to evaluate expressions involving integers. We will now use it to evaluate expressions involving fractions and mixed numbers.

1 Use the order of operations rule.

Recall from Section 1.9 that if we don't establish a uniform order of operations, an expression can have more than one value. To avoid this possibility, we must always use the following rule.

Order of Operations

1. Perform all calculations within parentheses and other grouping symbols following the order listed in Steps 2–4 below, working from the innermost pair of grouping symbols to the outermost pair.
2. Evaluate all exponential expressions.
3. Perform all multiplications and divisions as they occur from left to right.
4. Perform all additions and subtractions as they occur from left to right.

When grouping symbols have been removed, repeat Steps 2–4 to complete the calculation.

If a fraction bar is present, evaluate the expression above the bar (called the **numerator**) and the expression below the bar (called the **denominator**) separately. Then perform the division indicated by the fraction bar, if possible.

Self Check 1

Evaluate: $\frac{7}{8} + \frac{3}{2}\left(-\frac{1}{4}\right)^2$

Now Try **Problem 15**

EXAMPLE 1 Evaluate: $\frac{3}{4} + \frac{5}{3}\left(-\frac{1}{2}\right)^3$

Strategy We will scan the expression to determine what operations need to be performed. Then we will perform those operations, one-at-a-time, following the order of operations rule.

WHY If we don't follow the correct order of operations, the expression can have more than one value.

Solution

Although the expression contains parentheses, there are no calculations to perform *within* them. We will begin with step 2 of the rule: Evaluate all exponential expressions. We will write the steps of the solution in horizontal form.

$$\frac{3}{4} + \frac{5}{3}\left(-\frac{1}{2}\right)^3 = \frac{3}{4} + \frac{5}{3}\left(-\frac{1}{8}\right)$$ Evaluate: $\left(-\frac{1}{2}\right)^3 = \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1}{8}$.

$$= \frac{3}{4} + \left(-\frac{5}{24}\right)$$ Multiply: $\frac{5}{3}\left(-\frac{1}{8}\right) = -\frac{5 \cdot 1}{3 \cdot 8} = -\frac{5}{24}$.

$$= \frac{3}{4} \cdot \frac{6}{6} + \left(-\frac{5}{24}\right)$$ Prepare to add the fractions: Their LCD is 24. To build the first fraction so that its denominator is 24, multiply it by a form of 1.

$$= \frac{18}{24} + \left(-\frac{5}{24}\right)$$ Multiply the numerators: $3 \cdot 6 = 18$. Multiply the denominators: $4 \cdot 6 = 24$.

$$= \frac{13}{24}$$ Add the numerators: $18 + (-5) = 13$. Write the sum over the common denominator 24.

If an expression contains grouping symbols, we perform the operations within the grouping symbols first.

EXAMPLE 2 Evaluate: $\left(\frac{7}{8} - \frac{1}{4}\right) \div \left(-2\frac{3}{16}\right)$

Strategy We will perform any operations within parentheses first.

WHY This is the first step of the order of operations rule.

Solution

We will begin by performing the subtraction within the first set of parentheses. The second set of parentheses does not contain an operation to perform.

$$\left(\frac{7}{8} - \frac{1}{4}\right) \div \left(-2\frac{3}{16}\right)$$

$$= \left(\frac{7}{8} - \frac{1}{4} \cdot \frac{2}{2}\right) \div \left(-2\frac{3}{16}\right)$$ Within the first set of parentheses, prepare to subtract the fractions: Their LCD is 8. Build $\frac{1}{4}$ so that its denominator is 8.

$$= \left(\frac{7}{8} - \frac{2}{8}\right) \div \left(-2\frac{3}{16}\right)$$ Multiply the numerators: $1 \cdot 2 = 2$. Multiply the denominators: $4 \cdot 2 = 8$.

$$= \frac{5}{8} \div \left(-2\frac{3}{16}\right)$$ Subtract the numerators: $7 - 2 = 5$. Write the difference over the common denominator 8.

$$= \frac{5}{8} \div \left(-\frac{35}{16}\right)$$ Write the mixed number as an improper fraction.

$$= \frac{5}{8}\left(-\frac{16}{35}\right)$$ Use the rule for division of fractions: Multiply the first fraction by the reciprocal of $-\frac{35}{16}$.

$$= -\frac{5 \cdot 16}{8 \cdot 35}$$ Multiply the numerators and multiply the denominators. The product of two fractions with unlike signs is negative.

$$= -\frac{\overset{1}{\cancel{5}} \cdot 2 \cdot \overset{1}{\cancel{8}}}{\underset{1}{\cancel{8}} \cdot \underset{1}{\cancel{5}} \cdot 7}$$ To simplify, factor 16 as $2 \cdot 8$ and factor 35 as $5 \cdot 7$. Remove the common factors of 5 and 8 from the numerator and denominator.

$$= -\frac{2}{7}$$ Multiply the remaining factors in the numerator. Multipy the remaining factors in the denominator.

Self Check 2

Evaluate: $\left(\frac{19}{21} - \frac{2}{3}\right) \div \left(-2\frac{1}{7}\right)$

Now Try **Problem 19**

EXAMPLE 3 Add $7\frac{1}{3}$ to the difference of $\frac{5}{6}$ and $\frac{1}{4}$.

Strategy We will translate the words of the problem to numbers and symbols. Then we will use the order of operations rule to evaluate the resulting expression.

WHY Since the expression involves two operations, addition and subtraction, we need to perform them in the proper order.

Self Check 3

Add $2\frac{1}{4}$ to the difference of $\frac{7}{8}$ and $\frac{2}{3}$.

Now Try **Problem 23**

Solution

The key word *difference* indicates subtraction. Since we are to add $7\frac{1}{3}$ to the difference, the difference should be written first within parentheses, followed by the addition.

Add $7\frac{1}{3}$ to the difference of $\frac{5}{6}$ and $\frac{1}{4}$.

$$\left(\frac{5}{6} - \frac{1}{4}\right) + 7\frac{1}{3}$$ Translate from words to numbers and mathematical symbols.

$$\left(\frac{5}{6} - \frac{1}{4}\right) + 7\frac{1}{3} = \left(\frac{5}{6} \cdot \frac{2}{2} - \frac{1}{4} \cdot \frac{3}{3}\right) + 7\frac{1}{3}$$ Prepare to subtract the fractions within the parentheses. Build the fractions so that their denominators are the LCD 12.

$$= \left(\frac{10}{12} - \frac{3}{12}\right) + 7\frac{1}{3}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{7}{12} + 7\frac{1}{3}$$ Subtract the numerators: $10 - 3 = 7$. Write the difference over the common denominator 12.

$$= \frac{7}{12} + 7\frac{4}{12}$$ Prepare to add the fractions. Build $\frac{1}{3}$ so that its denominator is 12: $\frac{1}{3} \cdot \frac{4}{4} = \frac{4}{12}$.

$$= 7\frac{11}{12}$$ Add the numerators of the fractions: $7 + 4 = 11$. Write the sum over the common denominator 12.

2 Solve application problems by using the order of operations rule.

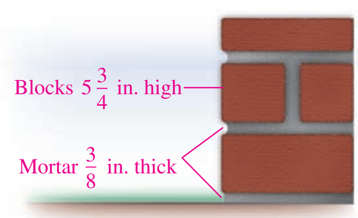
Sometimes more than one operation is needed to solve a problem.

Self Check 4

MASONRY Find the height of a wall if 8 layers (called *courses*) of $7\frac{3}{8}$-inch-high blocks are held together by $\frac{1}{4}$-inch-thick layers of mortar.

Now Try **Problem 77**

EXAMPLE 4 ***Masonry*** To build a wall, a mason will use blocks that are $5\frac{3}{4}$ inches high, held together with $\frac{3}{8}$-inch-thick layers of mortar. If the plans call for 8 layers, called *courses,* of blocks, what will be the height of the wall when completed?



Analyze

- The blocks are $5\frac{3}{4}$ inches high. *Given*
- A layer of mortar is $\frac{3}{8}$ inch thick. *Given*
- There are 8 layers (courses) of blocks. *Given*
- What is the height of the wall when completed? *Find*

Form To find the height of the wall when it is completed, we could add the heights of 8 blocks and 8 layers of mortar. However, it will be simpler if we find the height of one block and one layer of mortar, and multiply that result by 8.

The height of the wall when completed is equal to 8 times (the height of one block plus the thickness of one layer of mortar.)

The height of the wall when completed $= 8 \left(5\frac{3}{4} + \frac{3}{8}\right)$

Solve To evaluate the expression, we use the order of operations rule.

$$8\left(5\frac{3}{4}+\frac{3}{8}\right)=8\left(5\frac{6}{8}+\frac{3}{8}\right)$$

Prepare to add the fractions within the parentheses: Their LCD is 8. Build $\frac{3}{4}$ so that its denominator is 8: $\frac{3}{4}\cdot\frac{2}{2}=\frac{6}{8}$.

$$=8\left(5\frac{9}{8}\right)$$

Add the numerators of the fractions: $6+3=9$. Write the sum over the common denominator 8.

$$=\frac{8}{1}\left(\frac{49}{8}\right)$$

Prepare to multiply the fractions. Write $5\frac{9}{8}$ as an improper fraction.

$$=\frac{\overset{1}{\cancel{8}}\cdot 49}{1\cdot \underset{1}{\cancel{8}}}$$

Multiply the numerators and multiply the denominators. To simplify, remove the common factor of 8 from the numerator and denominator.

$$=49$$

Simplify: $\frac{49}{1}=49$.

State The completed wall will be 49 inches high.

Check We can estimate to check the result. Since one block and one layer of mortar is about 6 inches high, eight layers of blocks and mortar would be $8\cdot 6$ inches, or 48 inches high. The result of 49 inches seems reasonable.

3 Evaluate formulas.

To evaluate a formula, we replace its letters, called **variables**, with specific numbers and evaluate the right side using the order of operations rule.

EXAMPLE 5 The formula for the area of a trapezoid is $A=\frac{1}{2}h(a+b)$, where A is the area, h is the height, and a and b are the lengths of its bases. Find A when $h=1\frac{2}{3}$ in., $a=2\frac{1}{2}$ in., and $b=5\frac{1}{2}$ in.

Strategy In the formula, we will replace the letter h with $1\frac{2}{3}$, the letter a with $2\frac{1}{2}$, and the letter b with $5\frac{1}{2}$.

WHY Then we can use the order of operations rule to find the value of the expression on the right side of the = symbol.

Solution

$$A=\frac{1}{2}h(a+b)$$

This is the formula for the area of a trapezoid.

$$=\frac{1}{2}\left(1\frac{2}{3}\right)\left(2\frac{1}{2}+5\frac{1}{2}\right)$$

Replace h, a, and b with the given values.

$$=\frac{1}{2}\left(1\frac{2}{3}\right)(8)$$

Do the addition within the parentheses: $2\frac{1}{2}+5\frac{1}{2}=8$.

$$=\frac{1}{2}\left(\frac{5}{3}\right)\left(\frac{8}{1}\right)$$

To prepare to multiply fractions, write $1\frac{2}{3}$ as an improper fraction and 8 as $\frac{8}{1}$.

$$=\frac{1\cdot 5\cdot 8}{2\cdot 3\cdot 1}$$

Multiply the numerators. Multiply the denominators.

$$=\frac{1\cdot 5\cdot \overset{1}{\cancel{2}}\cdot 4}{\underset{1}{\cancel{2}}\cdot 3\cdot 1}$$

To simplify, factor 8 as $2\cdot 4$. Then remove the common factor of 2 from the numerator and denominator.

$$=\frac{20}{3}$$

Multiply the remaining factors in the numerator. Multiply the remaining factors in the denominator.

$$=6\frac{2}{3}$$

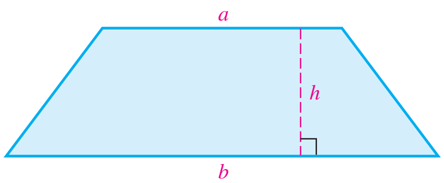
Write the improper fraction $\frac{20}{3}$ as a mixed number by dividing 20 by 3.

The area of the trapezoid is $6\frac{2}{3}$ in.^{2}.

Self Check 5

The formula for the area of a triangle is $A=\frac{1}{2}bh$. Find the area of a triangle whose base is $12\frac{1}{2}$ meters long and whose height is $15\frac{1}{3}$ meters.

Now Try **Problems 27 and 87**



A trapezoid

4 Simplify complex fractions.

Fractions whose numerators and/or denominators contain fractions are called *complex fractions*. Here is an example of a complex fraction:

A fraction in the numerator

$$\frac{\frac{3}{4}}{\frac{7}{8}}$$

The main fraction bar

A fraction in the denominator

Complex Fraction

A **complex fraction** is a fraction whose numerator or denominator, or both, contain one or more fractions or mixed numbers.

Here are more examples of complex fractions:

$$\frac{-\frac{1}{4}-\frac{4}{5}}{2\frac{4}{5}} \qquad \frac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{3}-\frac{1}{4}}$$

Numerator

Main fraction bar

Denominator

To *simplify* a complex fraction means to express it as a fraction in simplified form.

The following method for simplifying complex fractions is based on the fact that the main fraction bar indicates division.

$$\frac{\frac{1}{4}}{\frac{2}{5}} \longrightarrow \frac{1}{4} \div \frac{2}{5}$$

The main fraction bar means "divide the fraction in the numerator by the fraction in the denominator."

Simplifying a complex fraction

To simplify a complex fraction:

1. Add or subtract in the numerator and/or denominator so that the numerator is a single fraction and the denominator is a single fraction.
2. Perform the indicated division by multiplying the numerator of the complex fraction by the reciprocal of the denominator.
3. Simplify the result, if possible.

Self Check 6

Simplify: $\dfrac{\frac{1}{6}}{\frac{3}{8}}$

Now Try **Problem 31**

EXAMPLE 6

Simplify: $\dfrac{\frac{1}{4}}{\frac{2}{5}}$

Strategy We will perform the division indicated by the main fraction bar using the rule for dividing fractions from Section 3.3.

WHY We can skip step 1 and immediately divide because the numerator and the denominator of the complex fraction are already single fractions.

Solution

$$\frac{\frac{1}{4}}{\frac{2}{5}} = \frac{1}{4} \div \frac{2}{5}$$ Write the division indicated by the main fraction bar using a $\div$ symbol.

$$= \frac{1}{4} \cdot \frac{5}{2}$$ Use the rule for dividing fractions: Multiply the first fraction by the reciprocal of $\frac{2}{5}$, which is $\frac{5}{2}$.

$$= \frac{1 \cdot 5}{4 \cdot 2}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{5}{8}$$

EXAMPLE 7

Simplify: $\dfrac{-\frac{1}{4} + \frac{2}{5}}{\frac{1}{2} - \frac{4}{5}}$

Strategy Recall that a fraction bar is a type of grouping symbol. We will work above and below the main fraction bar separately to write $-\frac{1}{4} + \frac{2}{5}$ and $\frac{1}{2} - \frac{4}{5}$ as single fractions.

WHY The numerator and the denominator of the complex fraction must be written as single fractions before dividing.

Solution To write the numerator as a single fraction, we build $-\frac{1}{4}$ and $\frac{2}{5}$ to have an LCD of 20, and then add. To write the denominator as a single fraction, we build $\frac{1}{2}$ and $\frac{4}{5}$ to have an LCD of 10, and subtract.

$$\frac{-\frac{1}{4} + \frac{2}{5}}{\frac{1}{2} - \frac{4}{5}} = \frac{-\frac{1}{4} \cdot \frac{5}{5} + \frac{2}{5} \cdot \frac{4}{4}}{\frac{1}{2} \cdot \frac{5}{5} - \frac{4}{5} \cdot \frac{2}{2}}$$ The LCD for the numerator is 20. Build each fraction so that each has a denominator of 20. The LCD for the denominator is 10. Build each fraction so that each has a denominator of 10.

$$= \frac{-\frac{5}{20} + \frac{8}{20}}{\frac{5}{10} - \frac{8}{10}}$$ Multiply in the numerator. Multiply in the denominator.

$$= \frac{\frac{3}{20}}{-\frac{3}{10}}$$ In the numerator of the complex fraction, add the fractions. In the denominator, subtract the fractions.

$$= \frac{3}{20} \div \left(-\frac{3}{10}\right)$$ Write the division indicated by the main fraction bar using a $\div$ symbol.

$$= \frac{3}{20}\left(-\frac{10}{3}\right)$$ Multiply the first fraction by the reciprocal of $-\frac{3}{10}$, which is $-\frac{10}{3}$.

$$= -\frac{3 \cdot 10}{20 \cdot 3}$$ The product of two fractions with unlike signs is negative. Multiply the numerators. Multiply the denominators.

$$= -\frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{10}}}{2 \cdot \underset{1}{\cancel{10}} \cdot \underset{1}{\cancel{3}}}$$ To simplify, factor 20 as $2 \cdot 10$. Then remove the common factors of 3 and 10 from the numerator and denominator.

$$= -\frac{1}{2}$$ Multiply the remaining factors in the numerator. Multiply the remaining factors in the denominator.

Self Check 7

Simplify: $\dfrac{-\frac{5}{8} + \frac{1}{3}}{\frac{3}{4} - \frac{1}{3}}$

Now Try Problem 35

Self Check 8

Simplify: $\dfrac{5 - \frac{3}{4}}{1\frac{7}{8}}$

Now Try **Problem 39**

EXAMPLE 8

Simplify: $\dfrac{7 - \frac{2}{3}}{4\frac{5}{6}}$

Strategy Recall that a fraction bar is a type of grouping symbol. We will work above and below the main fraction bar separately to write $7 - \frac{2}{3}$ as a single fraction and $4\frac{5}{6}$ as an improper fraction.

WHY The numerator and the denominator of the complex fraction must be written as single fractions before dividing.

Solution

$$\frac{7 - \frac{2}{3}}{4\frac{5}{6}} = \frac{\frac{7}{1}\cdot\frac{3}{3} - \frac{2}{3}}{\frac{29}{6}}$$

In the numerator, write 7 as $\frac{7}{1}$. The LCD for the numerator is 3. Build $\frac{7}{1}$ so that it has a denominator of 3. In the denominator, write $4\frac{5}{6}$ as the improper fraction $\frac{29}{6}$.

$$= \frac{\frac{21}{3} - \frac{2}{3}}{\frac{29}{6}}$$

Multiply in the numerator.

$$= \frac{\frac{19}{3}}{\frac{29}{6}}$$

In the numerator of the complex fraction, subtract the numerators: $21 - 2 = 19$. Then write the difference over the common denominator 3.

$$= \frac{19}{3} \div \frac{29}{6}$$

Write the division indicated by the main fraction bar using a ÷ symbol.

$$= \frac{19}{3} \cdot \frac{6}{29}$$

Multiply the first fraction by the reciprocal of $\frac{29}{6}$, which is $\frac{6}{29}$.

$$= \frac{19 \cdot 6}{3 \cdot 29}$$

Multiply the numerators. Multiply the denominators.

$$= \frac{19 \cdot 2 \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \cdot 29}$$

To simplify, factor 6 as $2 \cdot 3$. Then remove the common factor of 3 from the numerator and denominator.

$$= \frac{38}{29}$$

Multiply the remaining factors in the numerator. Multiply the remaining factors in the denominator.

ANSWERS TO SELF CHECKS

1. $\frac{31}{32}$ **2.** $-\frac{1}{9}$ **3.** $2\frac{11}{24}$ **4.** 61 in. **5.** $95\frac{5}{6}$ m^2 **6.** $\frac{4}{9}$ **7.** $-\frac{7}{10}$ **8.** $\frac{34}{15}$

SECTION 3.7 STUDY SET

VOCABULARY

Fill in the blanks.

1. We use the order of __________ rule to evaluate expressions that contain more than one operation.

2. To evaluate a formula such as $A = \frac{1}{2}h(a + b)$, we substitute specific numbers for the letters, called __________, in the formula and find the value of the right side.

3. $\dfrac{\frac{1}{2}}{\frac{3}{4}}$ and $\dfrac{\frac{7}{8}+\frac{2}{5}}{\frac{1}{2}-\frac{1}{3}}$ are examples of ________ fractions.

4. In the complex fraction $\dfrac{\frac{2}{5}+\frac{1}{4}}{\frac{2}{5}-\frac{1}{4}}$, the __________ is $\frac{2}{5}+\frac{1}{4}$ and the ____________ is $\frac{2}{5}-\frac{1}{4}$.

CONCEPTS

5. What operations are involved in this expression?

$$5\left(6\frac{1}{3}\right)+\left(-\frac{1}{4}\right)^3$$

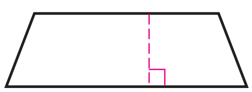
6. a. To evaluate $\frac{7}{8}+\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)$, what operation should be performed first?

b. To evaluate $\frac{7}{8}+\left(\frac{1}{3}-\frac{1}{4}\right)^2$, what operation should be performed first?

7. Translate the following to numbers and symbols. *You do not have to find the answer.*

Add $1\frac{2}{15}$ to the difference of $\frac{2}{3}$ and $\frac{1}{10}$.

8. Refer to the trapezoid shown below. Label the length of the upper base $3\frac{1}{2}$ inches, the length of the lower base $5\frac{1}{2}$ inches, and the height $2\frac{2}{3}$ inches.



9. What division is represented by this complex fraction?

$$\dfrac{\frac{2}{3}}{\frac{1}{5}}$$

10. Consider: $\dfrac{\frac{2}{3}-\frac{1}{5}}{\frac{1}{2}+\frac{4}{5}}$

a. What is the LCD for the fractions in the numerator of this complex fraction?

b. What is the LCD for the fractions in the denominator of this complex fraction?

11. Write the denominator of the following complex fraction as an improper fraction.

$$\dfrac{\frac{1}{8}-\frac{3}{16}}{5\frac{3}{4}}$$

12. When this complex fraction is simplified, will the result be positive or negative?

$$\dfrac{-\frac{2}{3}}{\frac{3}{4}}$$

NOTATION

Fill in the blanks to complete each solution.

13.
$$\begin{aligned}\frac{7}{12}-\frac{1}{2}\cdot\frac{1}{3}&=\frac{7}{12}-\frac{1\cdot 1}{2\cdot\square}\\&=\frac{7}{12}-\frac{1}{\square}\\&=\frac{7}{12}-\frac{1}{6}\cdot\frac{\square}{\square}\\&=\frac{7}{12}-\frac{\square}{12}\\&=\frac{\square}{12}\end{aligned}$$

14.
$$\begin{aligned}\dfrac{\frac{1}{8}}{\frac{3}{4}}&=\frac{1}{8}\div\frac{\square}{\square}\\&=\frac{1}{8}\cdot\frac{\square}{\square}\\&=\frac{1\cdot\square}{8\cdot 3}\\&=\frac{1\cdot\overset{1}{\cancel{\square}}}{2\cdot\underset{1}{\cancel{\square}}\cdot 3}\\&=\frac{1}{\square}\end{aligned}$$

GUIDED PRACTICE

Evaluate each expression. See Example 1.

15. $\frac{3}{4}+\frac{2}{5}\left(-\frac{1}{2}\right)^2$

16. $\frac{1}{4}+\frac{8}{27}\left(-\frac{3}{2}\right)^2$

17. $\frac{1}{6}+\frac{9}{8}\left(-\frac{2}{3}\right)^3$

18. $\frac{1}{5}+\frac{1}{9}\left(-\frac{3}{2}\right)^3$

Evaluate each expression. See Example 2.

19. $\left(\frac{3}{4} - \frac{1}{6}\right) \div \left(-2\frac{1}{6}\right)$

20. $\left(\frac{7}{8} - \frac{3}{7}\right) \div \left(-1\frac{3}{7}\right)$

21. $\left(\frac{15}{16} - \frac{1}{8}\right) \div \left(-9\frac{3}{4}\right)$

22. $\left(\frac{19}{36} - \frac{1}{6}\right) \div \left(-8\frac{2}{3}\right)$

Evaluate each expression. See Example 3.

23. Add $5\frac{4}{15}$ to the difference of $\frac{5}{6}$ and $\frac{2}{3}$.

24. Add $8\frac{5}{24}$ to the difference of $\frac{3}{4}$ and $\frac{1}{6}$.

25. Add $2\frac{7}{18}$ to the difference of $\frac{7}{9}$ and $\frac{1}{2}$.

26. Add $1\frac{19}{30}$ to the difference of $\frac{4}{5}$ and $\frac{1}{2}$.

Evaluate the formula $A = \frac{1}{2}h(a + b)$ for the given values. See Example 5.

27. $a = 2\frac{1}{2}, b = 7\frac{1}{2}, h = 5\frac{1}{4}$

28. $a = 4\frac{1}{2}, b = 5\frac{1}{2}, h = 2\frac{1}{8}$

29. $a = 1\frac{1}{4}, b = 6\frac{3}{4}, h = 4\frac{1}{2}$

30. $a = 1\frac{1}{3}, b = 4\frac{2}{3}, h = 2\frac{2}{5}$

Simplify each complex fraction. See Example 6.

31. $\dfrac{\frac{1}{16}}{\frac{2}{5}}$

32. $\dfrac{\frac{2}{11}}{\frac{3}{4}}$

33. $\dfrac{\frac{5}{8}}{\frac{3}{4}}$

34. $\dfrac{\frac{1}{5}}{\frac{8}{15}}$

Simplify each complex fraction. See Example 7.

35. $\dfrac{-\frac{1}{4} + \frac{2}{3}}{\frac{5}{6} + \frac{2}{3}}$

36. $\dfrac{-\frac{1}{2} + \frac{7}{8}}{\frac{3}{4} - \frac{1}{2}}$

37. $\dfrac{\frac{1}{3} - \frac{3}{4}}{\frac{1}{6} + \frac{2}{3}}$

38. $\dfrac{\frac{1}{3} - \frac{3}{4}}{\frac{1}{6} + \frac{1}{3}}$

Simplify each complex fraction. See Example 8.

39. $\dfrac{5 - \frac{5}{6}}{1\frac{1}{12}}$

40. $\dfrac{4 - \frac{3}{4}}{1\frac{7}{8}}$

41. $\dfrac{4 - \frac{7}{8}}{3\frac{1}{4}}$

42. $\dfrac{6 - \frac{2}{7}}{6\frac{2}{3}}$

TRY IT YOURSELF

Evaluate each expression and simplify each complex fraction.

43. $\frac{7}{8} - \left(\frac{4}{5} + 1\frac{3}{4}\right)$

44. $\left(\frac{5}{4}\right)^2 + \left(\frac{2}{3} - 2\frac{1}{6}\right)$

45. $\dfrac{-\frac{14}{15}}{\frac{7}{10}}$

46. $\dfrac{\frac{5}{27}}{-\frac{5}{9}}$

47. $A = \frac{1}{2}bh$ for $b = 10$ and $h = 7\frac{1}{5}$

48. $V = lwh$ for $l = 12$, $w = 8\frac{1}{2}$, and $h = 3\frac{1}{3}$

49. $\frac{2}{3}\left(-\frac{1}{4}\right) + \frac{1}{2}$

50. $-\frac{7}{8} - \left(\frac{1}{8}\right)\left(\frac{2}{3}\right)$

51. $\frac{4}{5} - \left(-\frac{1}{3}\right)^2$

52. $-\frac{3}{16} - \left(-\frac{1}{2}\right)^3$

53. $\dfrac{\frac{3}{8} + \frac{1}{4}}{\frac{3}{8} - \frac{1}{4}}$

54. $\dfrac{\frac{2}{5} + \frac{1}{4}}{\frac{2}{5} - \frac{1}{4}}$

55. Add $12\frac{11}{12}$ to the difference of $5\frac{1}{6}$ and $3\frac{7}{8}$.

56. Add $18\frac{1}{3}$ to the difference of $11\frac{3}{5}$ and $9\frac{11}{15}$.

57. $\dfrac{5\frac{1}{2}}{-\frac{1}{4}+\frac{3}{4}}$

58. $\dfrac{4\frac{1}{4}}{\frac{2}{3}+\left(-\frac{1}{6}\right)}$

59. $\left|\dfrac{2}{3}-\dfrac{9}{10}\right| \div \left(-\dfrac{1}{5}\right)$

60. $\left|-\dfrac{3}{16} \div 2\dfrac{1}{4}\right| + \left(-2\dfrac{1}{8}\right)$

61. $\dfrac{\frac{1}{5}-\left(-\frac{1}{4}\right)}{\frac{1}{4}+\frac{4}{5}}$

62. $\dfrac{\frac{1}{8}-\left(-\frac{1}{2}\right)}{\frac{1}{4}+\frac{3}{8}}$

63. $1\dfrac{3}{5}\left(\dfrac{1}{2}\right)^2\left(\dfrac{3}{4}\right)$

64. $2\dfrac{3}{5}\left(-\dfrac{1}{3}\right)^2\left(\dfrac{1}{2}\right)$

65. $A = lw$, for $l = 5\dfrac{5}{6}$ and $w = 7\frac{3}{5}$.

66. $P = 2l + 2w$, for $l = \dfrac{7}{8}$ and $w = \dfrac{3}{5}$.

67. $\left(2-\dfrac{1}{2}\right)^2 + \left(2+\dfrac{1}{2}\right)^2$

68. $\left(\dfrac{9}{20} \div 2\dfrac{2}{5}\right) + \left(\dfrac{3}{4}\right)^2$

69. $\dfrac{-\frac{5}{6}}{-1\frac{7}{8}}$

70. $\dfrac{-\frac{4}{3}}{-2\frac{5}{6}}$

71. Subtract $9\dfrac{1}{10}$ from the sum of $7\dfrac{3}{7}$ and $3\dfrac{1}{5}$.

72. Subtract $3\dfrac{2}{3}$ from the sum of $2\dfrac{5}{12}$ and $1\dfrac{5}{8}$.

73. $\dfrac{\frac{1}{2}+\frac{1}{4}}{\frac{1}{2}-\frac{1}{4}}$

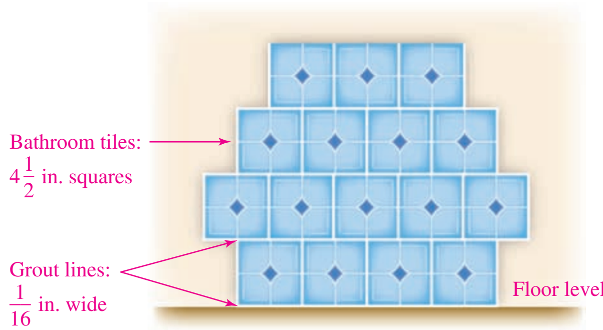
74. $\dfrac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{3}-\frac{1}{4}}$

75. $\left(\dfrac{8}{5}-1\dfrac{1}{3}\right)-\left(-\dfrac{4}{5}\cdot 10\right)$

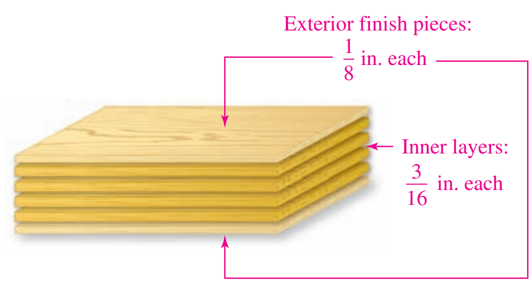
76. $\left(1-\dfrac{3}{4}\right)\left(1+\dfrac{3}{4}\right)$

APPLICATIONS

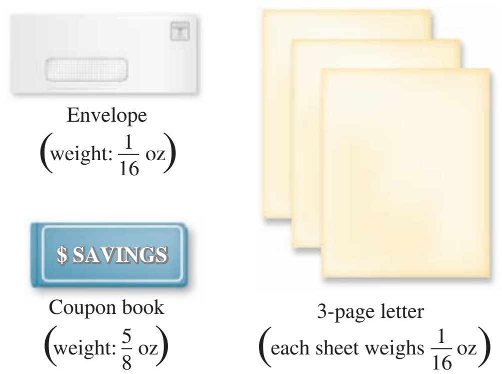
77. REMODELING A BATHROOM A handyman installed 20 rows of grout and tile on a bathroom wall using the pattern shown below. How high above floor level does the tile work reach? (*Hint:* There is no grout line above the last row of tiles.)



78. PLYWOOD To manufacture a sheet of plywood, several thin layers of wood are glued together, as shown. Then an exterior finish is attached to the top and the bottom, as shown below. How thick is the final product?



79. POSTAGE RATES Can the advertising package shown below be mailed for the 1-ounce rate?



80. PHYSICAL THERAPY After back surgery, a patient followed a walking program shown in the table below to strengthen her muscles. What was the total distance she walked over this three-week period?

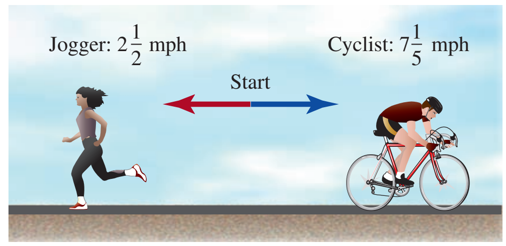
Week	Distance per day
#1	$\frac{1}{4}$ mile
#2	$\frac{1}{2}$ mile
#3	$\frac{3}{4}$ mile

81. READING PROGRAMS To improve reading skills, elementary school children read silently at the end of the school day for $\frac{1}{4}$ hour on Mondays and for $\frac{1}{2}$ hour on Fridays. For the month of January, how many total hours did the children read silently in class?

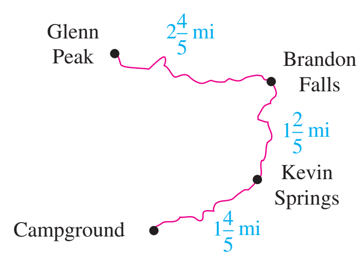
S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

82. PHYSICAL FITNESS Two people begin their workouts from the same point on a bike path and travel in opposite directions, as shown below. How far apart are they in $1\frac{1}{2}$ hours? Use the table to help organize your work.

	Rate (mph)	· Time (hr)	= Distance (mi)
Jogger			
Cyclist			



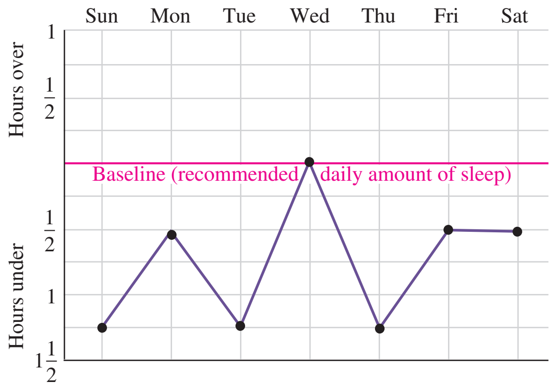
83. HIKING A scout troop plans to hike from the campground to Glenn Peak, as shown below. Since the terrain is steep, they plan to stop and rest after every $\frac{2}{3}$ mile. With this plan, how many parts will there be to this hike?



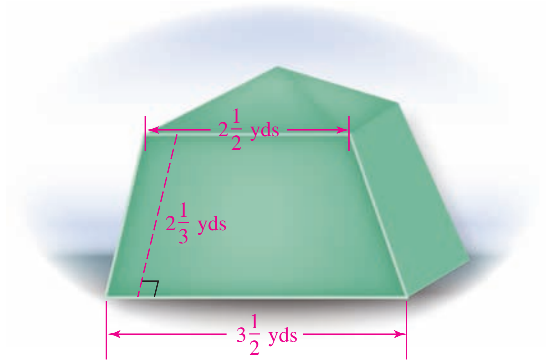
84. DELI SHOPS A sandwich shop sells a $\frac{1}{2}$-pound club sandwich made of turkey and ham. The owner buys the turkey in $1\frac{3}{4}$-pound packages and the ham in $2\frac{1}{2}$-pound packages. If he mixes two packages of turkey and one package of ham together, how many sandwiches can he make from the mixture?

85. SKIN CREAMS Using a formula of $\frac{1}{2}$ ounce of sun block, $\frac{2}{3}$ ounce of moisturizing cream, and $\frac{3}{4}$ ounce of lanolin, a beautician mixes her own brand of skin cream. She packages it in $\frac{1}{4}$-ounce tubes. How many full tubes can be produced using this formula? How much skin cream is left over?

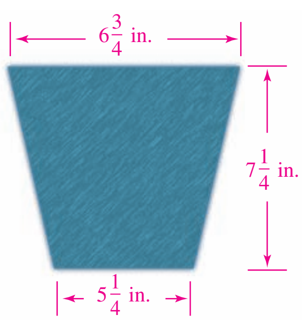
86. SLEEP The graph below compares the amount of sleep a 1-month-old baby got to the $15\frac{1}{2}$-hour daily requirement recommended by Children's Hospital of Orange County, California. For the week, how far below the baseline was the baby's daily average?

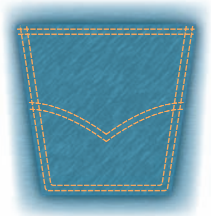


87. CAMPING The four sides of a tent are all the same trapezoid-shape. (See the illustration below.) How many square yards of canvas are used to make one of the sides of the tent?



88. SEWING A seamstress begins with a trapezoid-shaped piece of denim to make the back pocket on a pair of jeans. (See the illustration below.) How many square inches of denim are used to make the pocket?




Finished pocket

89. AMUSEMENT PARKS At the end of a ride at an amusement park, a boat splashes into a pool of water. The time (in seconds) that it takes two pipes to refill the pool is given by

$$\frac{1}{\frac{1}{10}+\frac{1}{15}}$$

Simplify the complex faction to find the time.

90. ALGEBRA Complex fractions, like the one shown below, are seen in an algebra class when the topic of *slope of a line* is studied. Simplify this complex fraction and, as is done in algebra, write the answer as an improper fraction.

$$\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{4}-\frac{1}{5}}$$

WRITING

91. Why is an order of operations rule necessary?

92. What does it mean to evaluate a formula?

93. What is a complex fraction?

94. In the complex fraction $\frac{\frac{3}{8}+\frac{1}{4}}{\frac{3}{8}-\frac{1}{4}}$, the fraction bar serves as a grouping symbol. Explain why this is so.

REVIEW

95. Find the sum: $8 + 19 + 124 + 2{,}097$

96. Subtract 879 from 1,023.

97. Multiply 879 by 23.

98. Divide 1,665 by 45.

99. List the factors of 24.

100. Find the prime factorization of 24.

STUDY SKILLS CHECKLIST

Working with Fractions

Before taking the test on Chapter 3, make sure that you have a solid understanding of the following methods for simplifying, multiplying, dividing, adding, and subtracting fractions. Put a checkmark in the box if you can answer "yes" to the statement.

☐ I know how to simplify fractions by factoring the numerator and denominator and then removing the common factors.

$$\frac{42}{50} = \frac{2 \cdot 3 \cdot 7}{2 \cdot 5 \cdot 7}$$
$$= \frac{\overset{1}{\cancel{2}} \cdot 3 \cdot 7}{\underset{1}{\cancel{2}} \cdot 5 \cdot 5}$$
$$= \frac{21}{25}$$

☐ When multiplying fractions, I know that it is important to factor and simplify first, before multiplying.

Factor and simplify first

$$\frac{15}{16} \cdot \frac{24}{35} = \frac{15 \cdot 24}{16 \cdot 35}$$
$$= \frac{3 \cdot \overset{1}{\cancel{5}} \cdot 3 \cdot \overset{1}{\cancel{8}}}{2 \cdot \underset{1}{\cancel{8}} \cdot \underset{1}{\cancel{5}} \cdot 7}$$

Don't multiply first

$$\frac{15}{16} \cdot \frac{24}{35} = \frac{15 \cdot 24}{16 \cdot 35}$$
$$= \frac{360}{560}$$

☐ To divide fractions, I know to multiply the first fraction by the reciprocal of the second fraction.

$$\frac{7}{8} \div \frac{23}{24} = \frac{7}{8} \cdot \frac{24}{23}$$

☐ I know that to add or subtract fractions, they must have a common denominator. To multiply or divide fractions, they **do not** need to have a common denominator.

Need an LCD

$$\frac{2}{3} + \frac{1}{5} \qquad \frac{9}{20} - \frac{7}{12}$$

Do not need an LCD

$$\frac{4}{7} \cdot \frac{2}{9} \qquad \frac{11}{40} \div \frac{5}{8}$$

☐ I know how to find the LCD of a set of fractions using one of the following methods.

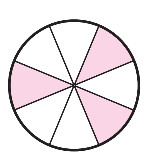
- Write the multiples of the largest denominator in increasing order, until one is found that is divisible by the other denominators.
- Prime factor each denominator. The LCM is a product of prime factors, where each factor is used the greatest number of times it appears in any one factorization.

☐ I know how to build equivalent fractions by multiplying the given fraction by a form of 1.

$$\frac{2}{3} = \frac{2}{3} \cdot \frac{5}{5}$$
$$= \frac{2 \cdot 5}{3 \cdot 5}$$
$$= \frac{10}{15}$$

CHAPTER 3 SUMMARY AND REVIEW

SECTION 3.1 An Introduction to Fractions

DEFINITIONS AND CONCEPTS	EXAMPLES
A **fraction** describes the number of equal parts of a whole. In a fraction, the number above the **fraction bar** is called the **numerator,** and the number below is called the **denominator.**	Since 3 of 8 equal parts are colored red, $\frac{3}{8}$ (three-eighths) of the figure is shaded.  Fraction bar → $\frac{3}{8}$ ← numerator denominator

If the numerator of a fraction is less than its denominator, the fraction is called a **proper fraction.** If the numerator of a fraction is greater than or equal to its denominator, the fraction is called an **improper fraction.**

Proper fractions: $\frac{1}{5}, \frac{7}{8}$, and $\frac{999}{1{,}000}$ — Proper fractions are less than 1.

Improper fractions: $\frac{3}{2}, \frac{41}{16}$, and $\frac{15}{15}$ — Improper fractions are greater than or equal to 1.

There are four **special fraction forms** that involve 0 and 1.

Each of these fractions is a **form of 1**:

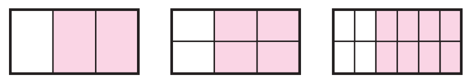
$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \frac{7}{7} = \frac{8}{8} = \frac{9}{9} = \ldots$$

Simplify each fraction:

$$\frac{0}{8} = 0 \qquad \frac{7}{0} \text{ is undefined} \qquad \frac{5}{1} = 5 \qquad \frac{20}{20} = 1$$

Two fractions are **equivalent** if they represent the same number. **Equivalent fractions** represent the same portion of a whole.

$\frac{2}{3}, \frac{4}{6}$, and $\frac{8}{12}$ are equivalent fractions. They represent the same shaded portion of the figure.



$$\frac{2}{3} = \frac{4}{6} = \frac{8}{12}$$

To **build a fraction,** we multiply it by a factor of 1 in the form $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}$, and so on.

Write $\frac{3}{4}$ as an equivalent fraction with a denominator of 36.

$$\frac{3}{4} = \frac{3}{4} \cdot \frac{9}{9}$$

We must multiply the denominator of $\frac{3}{4}$ by 9 to obtain a denominator of 36. It follows that $\frac{9}{9}$ should be the form of 1 that is used to build $\frac{3}{4}$.

$$= \frac{3 \cdot 9}{4 \cdot 9}$$

Multiply the numerators.
Multiply the denominators.

$$= \frac{27}{36}$$

$\frac{27}{36}$ is equivalent to $\frac{3}{4}$.

A fraction is **in simplest form,** or **lowest terms,** when the numerator and denominator have no common factors other than 1.

Is $\frac{6}{14}$ in simplest form?

The factors of the numerator, 6, are: **1**, **2**, 3, 6.
The factors of the denominator, 14, are: **1**, **2**, 7, 14.

Since the numerator and denominator have a common factor of 2, the fraction $\frac{6}{14}$ is *not* in simplest form.

To **simplify a fraction,** we write it in simplest form by removing a factor equal to 1:

1. Factor (or prime factor) the numerator and denominator to determine their common factors.
2. Remove factors equal to 1 by replacing each pair of factors common to the numerator and denominator with the equivalent fraction $\frac{1}{1}$.
3. Multiply the remaining factors in the numerator and in the denominator.

Simplify: $\frac{12}{30}$

$$\frac{12}{30} = \frac{2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 5}$$

Prime factor 12 and 30.

$$= \frac{\overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 5}$$

Remove the common factors of 2 and 3 from the numerator and denominator.

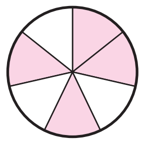
$$= \frac{2}{5}$$

Multiply the remaining factors in the numerator: $1 \cdot 2 \cdot 1 = 2$.
Multiply the remaining factors in the denominator: $1 \cdot 1 \cdot 5 = 5$.

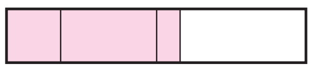
Since 2 and 5 have no common factors other than 1, we say that $\frac{2}{5}$ is in simplest form.

REVIEW EXERCISES

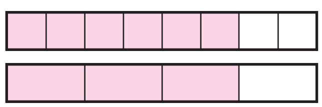
1. Identify the numerator and denominator of the fraction $\frac{11}{16}$. Is it a proper or an improper fraction?
2. Write fractions that represent the shaded and unshaded portions of the figure to the right.



3. In the illustration below, why can't we say that $\frac{3}{4}$ of the figure is shaded?



4. Write the fraction $\frac{2}{-3}$ in two other ways.
5. Simplify, if possible:
   a. $\frac{5}{5}$ b. $\frac{0}{10}$
   c. $\frac{18}{1}$ d. $\frac{7}{0}$
6. What concept about fractions is illustrated below?



Write each fraction as an equivalent fraction with the indicated denominator.

7. $\frac{2}{3}$, denominator 18
8. $\frac{3}{8}$, denominator 16
9. $\frac{7}{15}$, denominator 45
10. $\frac{13}{12}$, denominator 60
11. Write 5 as an equivalent fraction with denominator 9.
12. Are the following fractions in simplest form?
   a. $\frac{6}{9}$ b. $\frac{10}{81}$

Simplify each fraction, if possible.

13. $\frac{15}{45}$
14. $\frac{20}{48}$
15. $\frac{66}{108}$
16. $\frac{117}{208}$
17. $\frac{81}{64}$
18. Tell whether $\frac{8}{12}$ and $\frac{176}{264}$ are equivalent by simplifying each fraction.
19. SLEEP If a woman gets seven hours of sleep each night, write a fraction to describe the part of a whole day that she spends sleeping and another to describe the part of a whole day that she is not sleeping.
20. a. What type of problem is shown below? Explain the solution.

$$\frac{5}{8} = \frac{5}{8} \cdot \frac{2}{2} = \frac{10}{16}$$

   b. What type of problem is shown below? Explain the solution.

$$\frac{4}{6} = \frac{\overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{2}} \cdot 3} = \frac{2}{3}$$

SECTION 3.2 Multiplying Fractions

DEFINITIONS AND CONCEPTS	EXAMPLES
To **multiply two fractions,** multiply the numerators and multiply the denominators. Simplify the result, if possible.	Multiply and simplify, if possible: $\frac{4}{5} \cdot \frac{2}{3}$ $\frac{4}{5} \cdot \frac{2}{3} = \frac{4 \cdot 2}{5 \cdot 3}$ Multiply the numerators. Multiply the denominators. $= \frac{8}{15}$ Since 8 and 15 have no common factors other than 1, the result is in simplest form.

Multiplying signed fractions

The product of two fractions with the same (like) signs is positive. The product of two fractions with different (unlike) signs is negative.

Multiply and simplify, if possible: $-\frac{3}{4} \cdot \frac{2}{27}$

$$-\frac{3}{4} \cdot \frac{2}{27} = -\frac{3 \cdot 2}{4 \cdot 27}$$

Multiply the numerators.
Multiply the denominators.
Since the fractions have unlike signs, make the answer negative.

$$= -\frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot 3 \cdot 3}$$

Prime factor 4 and 27. Then simplify, by removing the common factors of 2 and 3 from the numerator and denominator.

$$= -\frac{1}{18}$$

Multiply the remaining factors in the numerator: $1 \cdot 1 = 1$.
Multiply the remaining factors in the denominator: $1 \cdot 2 \cdot 1 \cdot 3 \cdot 3 = 18$.

The base of an **exponential expression** can be a positive or a negative fraction.

The rule for multiplying two fractions can be extended to find the product of three or more fractions.

Evaluate: $\left(\frac{2}{3}\right)^3$

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$$

Write the base, $\frac{2}{3}$, as a factor 3 times.

$$= \frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3}$$

Multiply the numerators.
Multiply the denominators.

$$= \frac{8}{27}$$

This fraction is in simplified form.

When a **fraction is followed by the word *of*,** it indicates that we are to find a part of some quantity using multiplication.

To find $\frac{2}{5}$ of 35, we multiply:

$$\frac{2}{5} \textit{ of } 35 = \frac{2}{5} \cdot 35$$

The word *of* indicates multiplication.

$$= \frac{2}{5} \cdot \frac{35}{1}$$

Write 35 as a fraction: $35 = \frac{35}{1}$.

$$= \frac{2 \cdot 35}{5 \cdot 1}$$

Multiply the numerators.
Multiply the denominators.

$$= \frac{2 \cdot \overset{1}{\cancel{5}} \cdot 7}{\underset{1}{\cancel{5}} \cdot 1}$$

Prime factor 35. Then simplify by removing the common factor of 5 from the numerator and denominator.

$$= \frac{14}{1}$$

Multiply the remaining factors in the numerator and in the denominator.

$$= 14$$

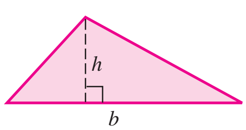
Any number divided by 1 is equal to that number.

The formula for the area of a triangle

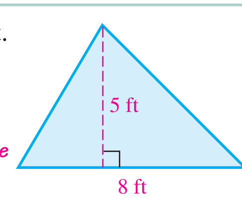
$$\text{Area of a triangle} = \frac{1}{2}(\text{base})(\text{height})$$

or

$$A = \frac{1}{2}bh$$



Find the area of the triangle shown on the right.



$$A = \frac{1}{2}(\textbf{base})(\textbf{height})$$

$$= \frac{1}{2}(8)(5)$$

Substitute 8 for the base and 5 for the height.

$$= \frac{1}{2}\left(\frac{5}{1}\right)\left(\frac{8}{1}\right)$$

Write 5 and 8 as fractions.

$$= \frac{1 \cdot 5 \cdot 8}{2 \cdot 1 \cdot 1}$$

Multiply the numerators.
Multiply the denominators.

$$= \frac{1 \cdot 5 \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 2}{\underset{1}{\cancel{2}} \cdot 1 \cdot 1}$$

Prime factor 8. Then simplify, by removing the common factor of 2 from the numerator and denominator.

$$= 20$$

The area of the triangle is 20 ft^2.

REVIEW EXERCISES

21. Fill in the blanks: To multiply two fractions, multiply the ___________ and multiply the _____________. Then _________, if possible.

22. Translate the following phrase to symbols. *You do not have to find the answer.*

$$\frac{5}{6} \text{ of } \frac{2}{3}$$

Multiply. Simplify the product, if possible.

23. $\frac{1}{2} \cdot \frac{1}{3}$

24. $\frac{2}{5}\left(-\frac{7}{9}\right)$

25. $\frac{9}{16} \cdot \frac{20}{27}$

26. $-\frac{5}{6}\left(-\frac{1}{15}\right)\left(-\frac{18}{25}\right)$

27. $\frac{3}{5} \cdot 7$

28. $-4\left(-\frac{9}{16}\right)$

29. $3\left(\frac{1}{3}\right)$

30. $-\frac{6}{7}\left(-\frac{7}{6}\right)$

Evaluate each expression.

31. $-\left(\frac{3}{4}\right)^2$

32. $\left(-\frac{5}{2}\right)^3$

33. $\left(-\frac{2}{5}\right)^3$

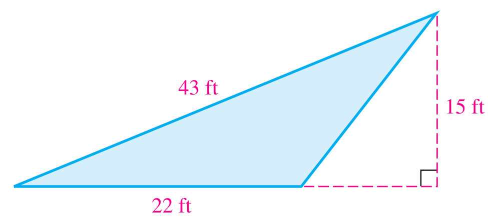
34. $\left(\frac{2}{3}\right)^2$

35. DRAG RACING A top-fuel dragster had to make 8 trial runs on a quarter-mile track before it was ready for competition. Find the total distance it covered on the trial runs.

36. GRAVITY Objects on the moon weigh only one-sixth of their weight on Earth. How much will an astronaut weigh on the moon if he weighs 180 pounds on Earth?

37. Find the area of the triangular sign.

38. Find the area of the triangle shown below.



SECTION 3.3 Dividing Fractions

DEFINITIONS AND CONCEPTS	EXAMPLES
One number is the **reciprocal** of another if their product is 1. To find the **reciprocal of a fraction,** invert the numerator and denominator.	The reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$ because $\frac{4}{5} \cdot \frac{5}{4} = 1$. Fraction Reciprocal $\frac{4}{5}$ $\frac{5}{4}$ Invert
To **divide two fractions,** multiply the first fraction by the reciprocal of the second fraction. Simplify the result, if possible.	Divide and simplify, if possible: $\frac{4}{35} \div \frac{2}{21}$ $\frac{4}{35} \div \mathbf{\frac{2}{21}} = \frac{4}{35} \cdot \mathbf{\frac{21}{2}}$ Multiply $\frac{4}{35}$ by the reciprocal of $\frac{2}{21}$, which is $\frac{21}{2}$. $= \frac{4 \cdot 21}{35 \cdot 2}$ Multiply the numerators. Multiply the denominators. $= \frac{2 \cdot 2 \cdot 3 \cdot 7}{5 \cdot 7 \cdot 2}$ To prepare to simplify, write 4, 21, and 35 in prime-factored form. $= \frac{\overset{1}{\cancel{2}} \cdot 2 \cdot 3 \cdot \overset{1}{\cancel{7}}}{5 \cdot \underset{1}{\cancel{7}} \cdot \underset{1}{\cancel{2}}}$ To simplify, remove the common factors of 2 and 7 from the numerator and denominator. $= \frac{6}{5}$ Multiply the remaining factors in the numerator: $1 \cdot 2 \cdot 3 \cdot 1 = 6$. Multiply the remaining factors in the denominator: $5 \cdot 1 \cdot 1 = 5$.

The **sign rules for dividing fractions** are the same as those for multiplying fractions.

Divide and simplify: $\frac{9}{16} \div (-3)$

$$\frac{9}{16} \div (-3) = \frac{9}{16} \cdot \left(-\frac{1}{3}\right)$$ Multiply $\frac{9}{16}$ by the reciprocal of -3, which is $-\frac{1}{3}$.

$$= -\frac{9 \cdot 1}{16 \cdot 3}$$ Multiply the numerators. Multiply the denominators. Since the fractions have unlike signs, make the answer negative.

$$= -\frac{\overset{1}{\cancel{3}} \cdot 3 \cdot 1}{16 \cdot \underset{1}{\cancel{3}}}$$ To simplify, factor 9 as $3 \cdot 3$. Then remove the common factor of 3 from the numerator and denominator.

$$= -\frac{3}{16}$$ Multiply the remaining factors in the numerator: $1 \cdot 3 \cdot 1 = 3$. Multiply the remaining factors in the denominator: $16 \cdot 1 = 16$.

Problems that involve forming **equal-sized groups** can be solved by division.

SEWING How many Halloween costumes, which require $\frac{3}{4}$ yard of material, can be made from 6 yards of material?

Since 6 yards of material is to be separated into an unknown number of equal-sized $\frac{3}{4}$-yard pieces, division is indicated.

$$6 \div \frac{3}{4} = \frac{6}{1} \cdot \frac{4}{3}$$ Write 6 as a fraction: $6 = \frac{6}{1}$. Multiply $\frac{6}{1}$ by the reciprocal of $\frac{3}{4}$, which is $\frac{4}{3}$.

$$= \frac{6 \cdot 4}{1 \cdot 3}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{2 \cdot \overset{1}{\cancel{3}} \cdot 4}{1 \cdot \underset{1}{\cancel{3}}}$$ To simplify, factor 6 as $2 \cdot 3$. Then remove the common factor of 3 from the numerator and denominator.

$$= \frac{8}{1}$$ Multiply the remaining factors in the numerator. Multiply the remaining factors in the denominator.

$$= 8$$ Any number divided by 1 is the same number.

The number of Halloween costumes that can be made from 6 yards of material is 8.

REVIEW EXERCISES

39. Find the reciprocal of each number.

a. $\frac{1}{8}$

b. $-\frac{11}{12}$

c. 5

d. $\frac{8}{7}$

40. Fill in the blanks: To divide two fractions, ________ the first fraction by the ________ of the second fraction.

Divide. Simplify the quotient, if possible.

41. $\frac{1}{6} \div \frac{11}{25}$

42. $-\frac{7}{32} \div \frac{1}{4}$

43. $-\frac{39}{25} \div \left(-\frac{13}{10}\right)$

44. $54 \div \frac{63}{5}$

45. $-\frac{3}{8} \div \frac{1}{4}$

46. $\frac{4}{5} \div \frac{1}{2}$

47. $\frac{2}{3} \div (-120)$

48. $\frac{7}{15} \div \frac{7}{15}$

49. MAKING JEWELRY How many $\frac{1}{16}$-ounce silver angel pins can be made from a $\frac{3}{4}$-ounce bar of silver?

50. SEWING How many pillow cases, which require $\frac{2}{3}$ yard of material, can be made from 20 yards of cotton cloth?

SECTION 3.4 Adding and Subtracting Fractions

DEFINITIONS AND CONCEPTS	EXAMPLES
To **add (or subtract) fractions that have the same denominator,** add (or subtract) the numerators and write the sum (or difference) over the common denominator. Simplify the result, if possible.	Add: $\frac{3}{16} + \frac{5}{16}$ $\frac{3}{16} + \frac{5}{16} = \frac{3+5}{16}$ Add the numerators and write the sum over the common denominator 16. $= \frac{8}{16}$ The resulting fraction can be simplified. $= \frac{\overset{1}{\cancel{8}}}{2 \cdot \underset{1}{\cancel{8}}}$ To simplify, factor 16 as 2 · 8. Then remove the common factor of 8 from the numerator and denominator. $= \frac{1}{2}$ Multiply the remaining factors in the denominator: 2 · 1 = 2.
Adding and subtracting fractions that have different denominators **1.** Find the LCD. **2.** Rewrite each fraction as an equivalent fraction with the LCD as the denominator. To do so, build each fraction using a form of 1 that involves any factors needed to obtain the LCD. **3.** Add or subtract the numerators and write the sum or difference over the LCD. **4.** Simplify the result, if possible.	Subtract: $\frac{4}{7} - \frac{1}{3}$ Since the smallest number the denominators 7 and 3 divide exactly is 21, the LCD is 21. $\frac{4}{7} - \frac{1}{3} = \frac{4}{7} \cdot \frac{3}{3} - \frac{1}{3} \cdot \frac{7}{7}$ To build $\frac{4}{7}$ and $\frac{1}{3}$ so that their denominators are 21, multiply each by a form of 1. $= \frac{12}{21} - \frac{7}{21}$ Multiply the numerators. Multiply the denominators. The denominators are now the same. $= \frac{12-7}{21}$ Subtract the numerators and write the difference over the common denominator 21. $= \frac{5}{21}$ This fraction is in simplest form.
The **least common denominator (LCD)** of a set of fractions is the **least common multiple (LCM)** of the denominators of the fractions. Two ways to find the LCM of the denominators are as follows: • Write the multiples of the largest denominator in increasing order, until one is found that is divisible by the other denominators. • Prime factor each denominator. The LCM is a product of prime factors, where each factor is used the greatest number of times it appears in any one factorization.	Add and simplify: $\frac{9}{20} + \frac{7}{15}$ To find the LCD, find the prime factorization of both denominators and use each prime factor the *greatest* number of times it appears in any one factorization: $\left.\begin{array}{l} 20 = 2 \cdot 2 \cdot 5 \\ 15 = 3 \cdot 5 \end{array}\right\}$ LCD $= 2 \cdot 2 \cdot 3 \cdot 5 = 60$ $\frac{9}{20} + \frac{7}{15} = \frac{9}{20} \cdot \frac{3}{3} + \frac{7}{15} \cdot \frac{4}{4}$ To build $\frac{9}{20}$ and $\frac{7}{15}$ so that their denominators are 60, multiply each by a form of 1. $= \frac{27}{60} + \frac{28}{60}$ Multiply the numerators. Multiply the denominators. The denominators are now the same. $= \frac{27+28}{60}$ Add the numerators and write the sum over the common denominator 60. $= \frac{55}{60}$ This fraction is not in simplest form. $= \frac{\overset{1}{\cancel{5}} \cdot 11}{2 \cdot 2 \cdot 3 \cdot \underset{1}{\cancel{5}}}$ To simplify, prime factor 55 and 60. Then remove the common factor of 5 from the numerator and denominator. $= \frac{11}{12}$ Multiply the remaining factors in the numerator and in the denominator.

Comparing fractions

If two fractions have the **same denominator,** the fraction with the greater numerator is the greater fraction.

If two fractions have **different denominators,** express each of them as an equivalent fraction that has the LCD for its denominator. Then compare numerators.

Which fraction is larger: $\frac{11}{18}$ or $\frac{7}{18}$?

$\frac{11}{18} > \frac{7}{18}$ because $11 > 7$

Which fraction is larger: $\frac{2}{3}$ or $\frac{3}{4}$?

Build each fraction to have a denominator that is the LCD, 12.

$$\frac{2}{3} = \frac{2}{3} \cdot \frac{4}{4} = \frac{8}{12} \qquad \frac{3}{4} = \frac{3}{4} \cdot \frac{3}{3} = \frac{9}{12}$$

Since $9 > 8$, it follows that $\frac{9}{12} > \frac{8}{12}$ and therefore, $\frac{3}{4} > \frac{2}{3}$.

REVIEW EXERCISES

Add or subtract and simplify, if possible.

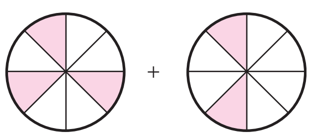
51. $\frac{2}{7} + \frac{3}{7}$

52. $\frac{3}{4} - \frac{1}{4}$

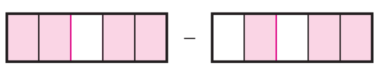
53. $\frac{7}{8} + \frac{3}{8}$

54. $-\frac{3}{5} - \frac{3}{5}$

55. **a.** Add the fractions represented by the figures below.



b. Subtract the fractions represented by the figures below.



56. Fill in the blanks. Use the prime factorizations below to find the least common denominator for fractions with denominators of 45 and 30.

$$\left.\begin{aligned} 45 &= 3 \cdot 3 \cdot 5 \\ 30 &= 2 \cdot 3 \cdot 5 \end{aligned}\right\} \text{LCD} = \square \cdot \square \cdot \square \cdot \square = \square$$

Add or subtract and simplify, if possible.

57. $\frac{1}{6} + \frac{2}{3}$

58. $-\frac{2}{5} - \frac{3}{8}$

59. $\frac{5}{24} + \frac{3}{16}$

60. $3 - \frac{1}{7}$

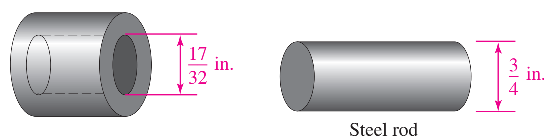
61. $-\frac{19}{18} + \frac{5}{12}$

62. $\frac{17}{20} - \frac{4}{15}$

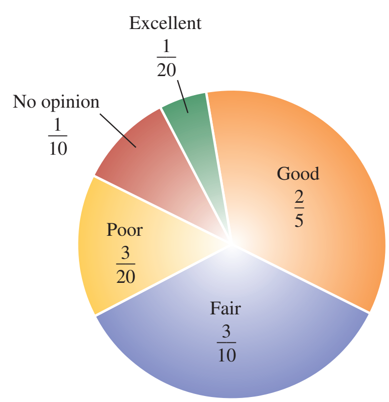
63. $-6 + \frac{13}{6}$

64. $\frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

65. MACHINE SHOPS How much must be milled off the $\frac{3}{4}$-inch-thick steel rod below so that the collar will slip over the end of it?



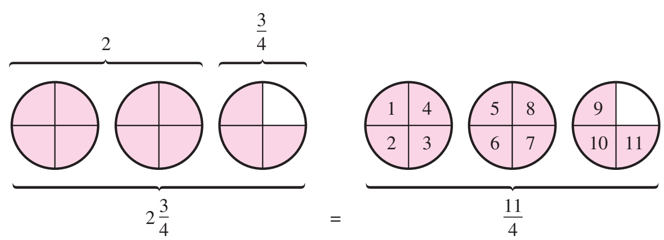
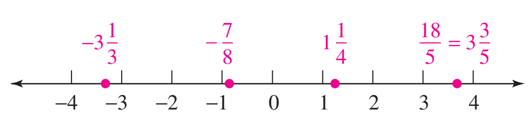
66. POLLS A group of adults were asked to rate the transportation system in their community. The results are shown below in a circle graph. What fraction of the group responded by saying either excellent, good, or fair?



67. TELEMARKETING In the first hour of work, a telemarketer made 2 sales out of 9 telephone calls. In the second hour, she made 3 sales out of 11 calls. During which hour was the rate of sales to calls better?

68. CAMERAS When the shutter of a camera stays open longer than $\frac{1}{125}$ second, any movement of the camera will probably blur the picture. With this in mind, if a photographer is taking a picture of a fast-moving object, should she select a shutter speed of $\frac{1}{60}$ or $\frac{1}{250}$?

SECTION 3.5 Multiplying and Dividing Mixed Numbers

DEFINITIONS AND CONCEPTS	EXAMPLES
A **mixed number** is the sum of a whole number and a proper fraction.	$\underbrace{2\frac{3}{4}}_{\text{Mixed number}} = \underbrace{2}_{\text{Whole-number part}} + \underbrace{\frac{3}{4}}_{\text{Fractional part}}$
There is a relationship between **mixed numbers and improper fractions** that can be seen using shaded regions.	Each disk represents one whole. 
To write a mixed number as an improper fraction: **1.** Multiply the denominator of the fraction by the whole-number part. **2.** Add the numerator of the fraction to the result from Step 1. **3.** Write the sum from Step 2 over the original denominator.	Write $3\frac{4}{5}$ as an improper fraction. Step 2: Add $3\frac{4}{5} = \frac{5 \cdot 3 + 4}{5} = \frac{15 + 4}{5} = \frac{19}{5}$ Step 1: Multiply Step 3: Use the same denominator From this result, it follows that $3\frac{4}{5} = \frac{19}{5}$.
To write an improper fraction as a mixed number: **1.** Divide the numerator by the denominator to obtain the whole-number part. **2.** The remainder over the divisor is the fractional part.	Write $\frac{47}{6}$ as mixed number. 7 ← The whole-number part is 7. $6\overline{)47}$ -42 5 ← Write the remainder 5 over the divisor 6 to get the fractional part. Thus, $\frac{47}{6} = 7\frac{5}{6}$. From this result, it follows that $\frac{47}{6} = 7\frac{5}{6}$.
Fractions and mixed numbers can be **graphed** on a number line.	Graph $-3\frac{1}{3}, 1\frac{1}{4}, \frac{18}{5}$, and $-\frac{7}{8}$ on a number line. 

To **multiply mixed numbers,** first change the mixed numbers to improper fractions. Then perform the multiplication of the fractions. Write the result as a mixed number or whole number in simplest form.

Multiply and simplify: $10\frac{1}{2} \cdot 1\frac{1}{6}$

$$10\frac{1}{2} \cdot 1\frac{1}{6} = \frac{21}{2} \cdot \frac{7}{6}$$
Write $10\frac{1}{2}$ and $1\frac{1}{6}$ as improper fractions.

$$= \frac{21 \cdot 7}{2 \cdot 6}$$
Use the rule for multiplying two fractions. Multiply the numerators. Multiply the denominators.

$$= \frac{\overset{1}{\cancel{3}} \cdot 7 \cdot 7}{2 \cdot 2 \cdot \underset{1}{\cancel{3}}}$$
To simplify, factor 21 as $3 \cdot 7$, and then remove the common factor of 3 from the numerator and denominator.

$$= \frac{49}{4}$$
Multiply the remaining factors in the numerator and in the denominator. The result is an improper fraction.

$$= 12\frac{1}{4}$$
Write the improper fraction $\frac{49}{4}$ as a mixed number.

$$\begin{array}{r} 12 \\ 4\overline{)49} \\ \underline{-4} \\ 09 \\ \underline{-8} \\ 1 \end{array}$$

To **divide mixed numbers,** first change the mixed numbers to improper fractions. Then perform the division of the fractions. Write the result as a mixed number or whole number in simplest form.

Divide and simplify: $5\frac{2}{3} \div \left(-3\frac{7}{9}\right)$

$$5\frac{2}{3} \div \left(-3\frac{7}{9}\right) = \frac{17}{3} \div \left(-\frac{34}{9}\right)$$
Write $5\frac{2}{3}$ and $3\frac{7}{9}$ as improper fractions.

$$= \frac{17}{3}\left(-\frac{9}{34}\right)$$
Multiply $\frac{17}{3}$ by the reciprocal of $-\frac{34}{9}$, which is $-\frac{9}{34}$.

$$= -\frac{17 \cdot 9}{3 \cdot 34}$$
Multiply the numerators. Multiply the denominators.

Since the fractions have unlike signs, make the answer negative.

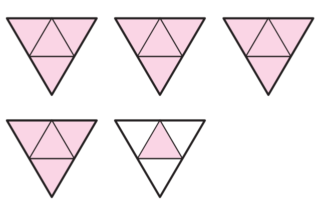
$$= -\frac{\overset{1}{\cancel{17}} \cdot \overset{1}{\cancel{3}} \cdot 3}{\underset{1}{\cancel{3}} \cdot 2 \cdot \underset{1}{\cancel{17}}}$$
To simplify, factor 9 as $3 \cdot 3$ and 34 as $2 \cdot 17$. Then remove the common factors of 3 and 17 from the numerator and denominator.

$$= -\frac{3}{2}$$
Multiply the remaining factors in the numerator and in the denominator. The result is a negative improper fraction.

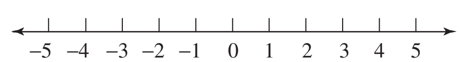
$$= -1\frac{1}{2}$$
Write the negative improper fraction $-\frac{3}{2}$ as a negative mixed number.

REVIEW EXERCISES

69. In the illustration below, each triangular region outlined in black represents one whole. Write a mixed number and an improper fraction to represent what is shaded.



70. Graph $-2\frac{2}{3}, \frac{8}{9}, -\frac{3}{4}$, and $\frac{59}{24}$ on a number line.



Write each improper fraction as a mixed number or a whole number.

71. $\frac{16}{5}$

72. $-\frac{47}{12}$

73. $\frac{51}{3}$

74. $\frac{14}{6}$

Write each mixed number as an improper fraction.

75. $9\frac{3}{8}$

76. $-2\frac{1}{5}$

77. $3\frac{11}{14}$

78. $1\frac{99}{100}$

Multiply or divide and simplify, if possible.

79. $1\frac{2}{5} \cdot 1\frac{1}{2}$

80. $-3\frac{1}{2} \div 3\frac{2}{3}$

81. $-6\left(-6\frac{2}{3}\right)$

82. $8 \div 3\frac{1}{5}$

83. $-11\frac{1}{5} \div \left(-\frac{7}{10}\right)$

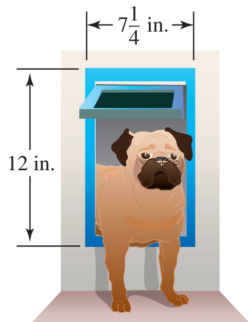
84. $5\frac{2}{3}\left(-7\frac{1}{5}\right)$

85. $\left(-2\frac{3}{4}\right)^2$

86. $1\frac{5}{16} \cdot 1\frac{7}{9} \cdot 2\frac{2}{3}$

87. PHOTOGRAPHY Each leg of a camera tripod can be extended to become $5\frac{1}{2}$ times its original length. If a leg is originally $8\frac{3}{4}$ inches long, how long will it become when it is completely extended?

88. PET DOORS Find the area of the opening provided by the rectangular-shaped pet door shown below.



89. PRINTING It takes a color copier $2\frac{1}{4}$ minutes to print a movie poster. How many posters can be printed in 90 minutes?

90. STORM DAMAGE A truck can haul $7\frac{1}{2}$ tons of trash in one load. How many loads would it take to haul away $67\frac{1}{2}$ tons from a hurricane cleanup site?

SECTION 3.6 Adding and Subtracting Mixed Numbers

DEFINITIONS AND CONCEPTS	EXAMPLES
To add (or subtract) mixed numbers, we can change each to an improper fraction and use the method of Section 3.4.	Add: $3\frac{1}{2} + 1\frac{3}{5}$
	$3\frac{1}{2} + 1\frac{3}{5} = \frac{7}{2} + \frac{8}{5}$ — Write $3\frac{1}{2}$ and $1\frac{3}{5}$ as mixed numbers.
	$= \frac{7}{2} \cdot \frac{5}{5} + \frac{8}{5} \cdot \frac{2}{2}$ — To build $\frac{7}{2}$ and $\frac{8}{5}$ so that their denominators are 10, multiply both by a form of 1.
	$= \frac{35}{10} + \frac{16}{10}$ — Multiply the numerators. Multiply the denominators.
	$= \frac{51}{10}$ — Add the numerators and write the sum over the common denominator 10.
	$= 5\frac{1}{10}$ — To write the improper fraction $\frac{51}{10}$ as a mixed number, divide 51 by 10.

To add (or subtract) mixed numbers, we can also write them in **vertical form** and add (or subtract) the whole-number parts and the fractional parts separately.	Add: $42\frac{1}{3} + 89\frac{6}{7}$ Build to get the LCD, 21. Add the fractions. Add the whole numbers. $\begin{array}{rcrcrcr} 42\frac{1}{3} & = & 42\frac{1}{3}\cdot\frac{7}{7} & = & 42\frac{7}{21} & = & \overset{1}{42}\frac{7}{21} \\ +\ 89\frac{6}{7} & = & +\ 89\frac{6}{7}\cdot\frac{3}{3} & = & +\ 89\frac{18}{21} & = & +\ 89\frac{18}{21} \\ \hline & & & & \frac{25}{21} & & 131\frac{25}{21} \end{array}$
When we add mixed numbers, sometimes the sum of the fractions is an improper fraction. If that is the case, write the improper fraction as a mixed number and **carry** its whole-number part to the whole-number column.	We don't want an improper fraction in the answer. Write $\frac{25}{21}$ as $1\frac{4}{21}$, carry the 1 to the whole-number column, and add it to 131 to get 132: $131\frac{25}{21} = 131 + 1\frac{4}{21} = 132\frac{4}{21}$
Subtraction of mixed numbers in vertical form sometimes involves **borrowing.** When the fraction we are subtracting is greater than the fraction we are subtracting it from, borrowing is necessary.	Subtract: $23\frac{1}{4} - 17\frac{5}{9}$ Build to get the LCD, 36. Since $\frac{20}{36}$ is greater than $\frac{9}{36}$, we must borrow from 28. $\begin{array}{rcrcrcrcr} 28\frac{1}{4} & = & 28\frac{1}{4}\cdot\frac{9}{9} & = & 28\frac{9}{36} & = & \overset{7}{\cancel{28}}\frac{9}{36} + \frac{36}{36} & = & \overset{7}{\cancel{28}}\frac{45}{36} \\ -\ 17\frac{5}{9} & = & -\ 17\frac{5}{9}\cdot\frac{4}{4} & = & -\ 17\frac{20}{36} & = & -\ 17\frac{20}{36} & = & -\ 17\frac{20}{36} \\ \hline & & & & & & & & 10\frac{25}{36} \end{array}$

REVIEW EXERCISES

Add or subtract and simplify, if possible.

91. $1\frac{3}{8} + 2\frac{1}{5}$

92. $3\frac{1}{2} + 2\frac{2}{3}$

93. $2\frac{5}{6} - 1\frac{3}{4}$

94. $3\frac{7}{16} - 2\frac{1}{8}$

95. $157\frac{11}{30} + 98\frac{7}{12}$

96. $6\frac{3}{14} + 17\frac{7}{10}$

97. $33\frac{8}{9} + 49\frac{1}{6}$

98. $98\frac{11}{20} + 14\frac{4}{5}$

99. $50\frac{5}{8} - 19\frac{1}{6}$

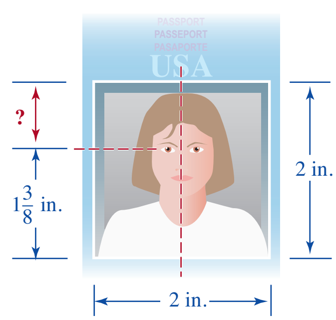
100. $375\frac{3}{4} - 59$

101. $23\frac{1}{3} - 2\frac{5}{6}$

102. $39 - 4\frac{5}{8}$

103. PAINTING SUPPLIES In a project to restore a house, painters used $10\frac{3}{4}$ gallons of primer, $21\frac{1}{2}$ gallons of latex paint, and $7\frac{2}{3}$ gallons of enamel. Find the total number of gallons of paint used.

104. PASSPORTS The required dimensions for a passport photograph are shown below. What is the distance from the subject's eyes to the top of the photograph?



SECTION 3.7 Order of Operations and Complex Fractions

DEFINITIONS AND CONCEPTS	EXAMPLES

Order of Operations

1. Perform all calculations within parentheses and other grouping symbols following the order listed in Steps 2–4 below, working from the innermost pair of grouping symbols to the outermost pair.
2. Evaluate all exponential expressions.
3. Perform all multiplications and divisions as they occur from left to right.
4. Perform all additions and subtractions as they occur from left to right.

When grouping symbols have been removed, repeat Steps 2–4 to complete the calculation.

If a fraction bar is present, evaluate the expression above the bar (called the **numerator**) and the expression below the bar (called the **denominator**) separately. Then perform the division indicated by the fraction bar, if possible.

Evaluate: $\left(\frac{1}{3}\right)^2 \div \left(\frac{3}{4} - \frac{1}{3}\right)$

First, we perform the subtraction within the second set of parentheses. (There is no operation to perform within the first set.)

$$\left(\frac{1}{3}\right)^2 \div \left(\frac{3}{4} - \frac{1}{3}\right)$$

$$= \left(\frac{1}{3}\right)^2 \div \left(\frac{3}{4}\cdot\frac{3}{3} - \frac{1}{3}\cdot\frac{4}{4}\right)$$ Within the parentheses, build each fraction so that its denominator is the LCD 12.

$$= \left(\frac{1}{3}\right)^2 \div \left(\frac{9}{12} - \frac{4}{12}\right)$$ Multiply the numerators. Multiply the denominators.

$$= \left(\frac{1}{3}\right)^2 \div \frac{5}{12}$$ Subtract the numerators: $9 - 4 = 5$. Write the difference over the common denominator 12.

$$= \frac{1}{9} \div \frac{5}{12}$$ Evaluate the exponential expression: $\left(\frac{1}{3}\right)^2 = \frac{1}{3}\cdot\frac{1}{3} = \frac{1}{9}$.

$$= \frac{1}{9}\cdot\frac{12}{5}$$ Use the rule for dividing fractions: Multiply the first fraction by the reciprocal of $\frac{5}{12}$, which is $\frac{12}{5}$.

$$= \frac{1 \cdot 12}{9 \cdot 5}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{1 \cdot \overset{1}{\cancel{3}} \cdot 4}{\underset{1}{\cancel{3}} \cdot 3 \cdot 5}$$ To simplify, factor 12 as $3 \cdot 4$ and 9 as $3 \cdot 3$. Then remove the common factor of 3 from the numerator and denominator.

$$= \frac{4}{15}$$ Multiply the remaining factors in the numerator. Multiply the remaining factors in the denominator.

To **evaluate a formula,** we replace its variables (letters) with specific numbers and evaluate the right side using the order of operations rule.

Evaluate: $A = \frac{1}{2}h(a + b)$ for $a = 1\frac{1}{3}$, $b = 2\frac{2}{3}$, and $h = 2\frac{4}{5}$.

$$A = \frac{1}{2}h(a + b)$$ This is the given formula.

$$= \frac{1}{2}\left(2\frac{4}{5}\right)\left(1\frac{1}{3} + 2\frac{2}{3}\right)$$ Replace *h*, *a*, and *b* with the given values.

$$= \frac{1}{2}\left(2\frac{4}{5}\right)(4)$$ Do the addition within the parentheses.

$$= \frac{1}{2}\left(\frac{14}{5}\right)\left(\frac{4}{1}\right)$$ To prepare to multiply fractions, write $2\frac{4}{5}$ as an improper fraction and 4 as $\frac{4}{1}$.

$$= \frac{1 \cdot 14 \cdot 4}{2 \cdot 5 \cdot 1}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{1 \cdot 14 \cdot \overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{2}} \cdot 5 \cdot 1}$$ To simplify, factor 4 as $2 \cdot 2$. Then remove the common factor of 2 from the numerator and denominator.

$$= \frac{28}{5}$$ Multiply the remaining factors in the numerator. Multiply the remaining factors in the denominator.

$$= 5\frac{3}{5}$$ Write the improper fraction $\frac{28}{5}$ as a mixed number by dividing 28 by 5.

A **complex fraction** is a fraction whose numerator or denominator, or both, contain one or more fractions or mixed numbers.

Complex fractions:

$$\frac{\frac{9}{10}}{\frac{27}{5}} \qquad \frac{\frac{2}{5}-\frac{1}{3}}{\frac{3}{7}+\frac{1}{5}} \qquad \frac{-7\frac{1}{4}}{2-\frac{1}{9}}$$

The method for **simplifying complex fractions** is based on the fact that the main fraction bar indicates division.

Simplify: $\dfrac{\frac{9}{10}}{\frac{27}{5}}$

$$\frac{\frac{9}{10}}{\frac{27}{5}} = \frac{9}{10} \div \frac{27}{5}$$ Write the division indicated by the main fraction bar using a ÷ symbol.

$$= \frac{9}{10} \cdot \frac{5}{27}$$ Use the rule for dividing fractions: Multiply the first fraction by the reciprocal of $\frac{27}{5}$, which is $\frac{5}{27}$.

$$= \frac{9 \cdot 5}{10 \cdot 27}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{\overset{1}{\cancel{9}} \cdot \overset{1}{\cancel{5}}}{2 \cdot \underset{1}{\cancel{5}} \cdot 3 \cdot \underset{1}{\cancel{9}}}$$ To simplify, factor 10 as $2 \cdot 5$ and 27 as $3 \cdot 9$. Then remove the common factors of 9 and 5 from the numerator and denominator.

$$= \frac{1}{6}$$ Multiply the remaining factors in the numerator. Multiply the remaining factors in the denominator.

To **simplify a complex fraction:**

1. Add or subtract in the numerator and/or denominator so that the numerator is a single fraction and the denominator is a single fraction.
2. Perform the indicated division by multiplying the numerator of the complex fraction by the reciprocal of the denominator.
3. Simplify the result, if possible.

Simplify: $\dfrac{\frac{2}{5}-\frac{1}{3}}{\frac{3}{7}+\frac{1}{5}}$

$$\frac{\frac{2}{5}-\frac{1}{3}}{\frac{3}{7}+\frac{1}{5}} = \frac{\frac{2}{5}\cdot\frac{3}{3}-\frac{1}{3}\cdot\frac{5}{5}}{\frac{3}{7}\cdot\frac{5}{5}+\frac{1}{5}\cdot\frac{7}{7}}$$ In the numerator, build each fraction so that each has a denominator of 15. In the denominator, build each fraction so that each has a denominator of 35.

$$= \frac{\frac{6}{15}-\frac{5}{15}}{\frac{15}{35}+\frac{7}{35}}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{\frac{1}{15}}{\frac{22}{35}}$$ Subtract the numerators and write the difference over the common denominator 15. Add the numerators and write the sum over the common denominator 35.

$$= \frac{1}{15} \div \frac{22}{35}$$ Write the division indicated by the main fraction bar using a ÷ symbol.

$$= \frac{1}{15} \cdot \frac{35}{22}$$ Use the rule for dividing fractions: Multiply the first fraction by the reciprocal of $\frac{22}{35}$, which is $\frac{35}{22}$.

$$= \frac{1 \cdot 35}{15 \cdot 22}$$ Multiply the numerators. Multiply the denominators.

$$= \frac{1 \cdot \overset{1}{\cancel{5}} \cdot 7}{3 \cdot \underset{1}{\cancel{5}} \cdot 22}$$ To simplify, factor 35 as $5 \cdot 7$ and 15 as $3 \cdot 5$. Then remove the common factor of 5 from the numerator and denominator.

$$= \frac{7}{66}$$ Multiply the remaining factors in the numerator. Multiply the remaining factors in the denominator.

REVIEW EXERCISES

Evaluate each expression.

105. $\frac{3}{4} + \left(-\frac{1}{3}\right)^2\left(\frac{5}{4}\right)$

106. $\left(\frac{2}{3} \div \frac{16}{9}\right) - \left(1\frac{2}{3} \cdot \frac{1}{15}\right)$

107. $\left(\frac{11}{5} - 1\frac{2}{3}\right) - \left(-\frac{4}{9} \cdot 18\right)$

108. $\left|-\frac{9}{16} \div 2\frac{1}{4}\right| + \left(-3\frac{7}{8}\right)$

Simplify each complex fraction.

109. $\dfrac{\frac{3}{5}}{-\frac{17}{20}}$

110. $\dfrac{4 - \frac{2}{7}}{4\frac{1}{7}}$

111. $\dfrac{\frac{2}{3} - \frac{1}{6}}{-\frac{3}{4} - \frac{1}{2}}$

112. $\dfrac{5\frac{1}{4}}{\frac{7}{4} + \left(-\frac{1}{3}\right)}$

113. Subtract $4\frac{1}{8}$ from the sum of $5\frac{1}{5}$ and $1\frac{1}{2}$.

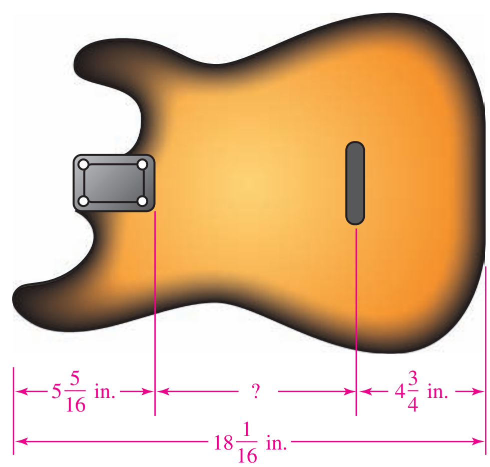
114. Add $12\frac{11}{16}$ to the difference of $4\frac{5}{8}$ and $3\frac{1}{4}$.

115. Evaluate the formula $A = \frac{1}{2}h(a + b)$ for $a = 1\frac{1}{8}$, $b = 4\frac{7}{8}$, and $h = 2\frac{7}{9}$.

116. Evaluate the formula $P = 2\ell + 2w$ for $\ell = 2\frac{1}{3}$ and $w = 3\frac{1}{4}$.

117. DERMATOLOGY A dermatologist mixes $1\frac{1}{2}$ ounces of cucumber extract, $2\frac{2}{3}$ ounces of aloe vera cream, and $\frac{3}{4}$ ounce of vegetable glycerin to make his own brand of anti-wrinkle cream. He packages it in $\frac{5}{6}$-ounce tubes. How many full tubes can be produced using this formula? How much cream is left over?

118. GUITAR DESIGN Find the missing dimension on the vintage 1962 Stratocaster body shown below.



CHAPTER 3 TEST

1. Fill in the blanks.

a. For the fraction $\frac{6}{7}$, the __________ is 6 and the __________ is 7.

b. Two fractions are __________ if they represent the same number.

c. A fraction is in ________ form when the numerator and denominator have no common factors other than 1.

d. To ________ a fraction, we remove common factors of the numerator and denominator.

e. The __________ of $\frac{4}{5}$ is $\frac{5}{4}$.

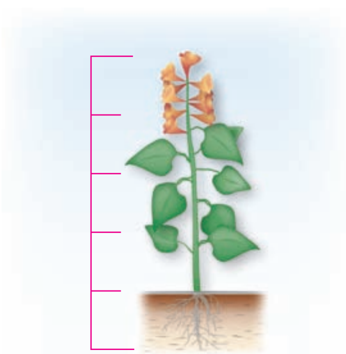
f. A ______ number, such as $1\frac{9}{16}$, is the sum of a whole number and a proper fraction.

g. $\dfrac{\frac{1}{8}}{\frac{7}{12}}$ and $\dfrac{\frac{3}{4}+\frac{1}{3}}{\frac{5}{12}-\frac{1}{4}}$ are examples of ________ fractions.

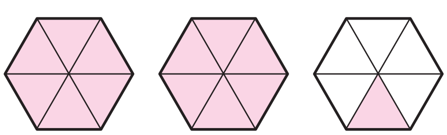
2. See the illustration below.

a. What fractional part of the plant is above ground?

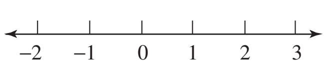
b. What fractional part of the plant is below ground?



3. Each region outlined in black represents one whole. Write an improper fraction and a mixed number to represent the shaded portion.



4. Graph $2\frac{4}{5}, -\frac{2}{5}, -1\frac{1}{7}$, and $\frac{7}{6}$ on a number line.



5. Are $\frac{1}{3}$ and $\frac{5}{15}$ equivalent?

6. Express $\frac{7}{8}$ as an equivalent fraction with denominator 24.

7. Simplify each fraction, if possible.

a. $\frac{0}{15}$ **b.** $\frac{9}{0}$

8. Simplify each fraction.

a. $\frac{27}{36}$ **b.** $\frac{72}{180}$

9. Add and simplify, if possible: $\frac{3}{16}+\frac{7}{16}$

10. Multiply and simplify, if possible: $-\frac{3}{4}\left(\frac{1}{5}\right)$

11. Divide and simplify, if possible: $\frac{4}{3} \div \frac{2}{9}$

12. Subtract and simplify, if possible: $\frac{11}{12}-\frac{11}{30}$

13. Add and simplify, if possible: $-\frac{3}{7}+2$

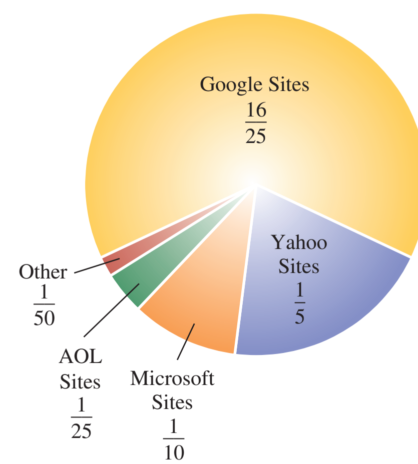
14. Multiply and simplify, if possible: $\frac{9}{10}\left(-\frac{4}{15}\right)\left(-\frac{25}{18}\right)$

15. Which fraction is larger: $\frac{8}{9}$ or $\frac{9}{10}$?

16. COFFEE DRINKERS Two-fifths of 100 adults surveyed said they started their morning with a cup of coffee. Of the 100, how many would this be?

17. THE INTERNET The graph below shows the fraction of the total number of Internet searches that were made using various sites in January 2009. What fraction of the all the searches were done using Google, Yahoo, or Microsoft sites?

Online Search Share
January 2009



Source: Marketingcharts.com

18. **a.** Write $\frac{55}{6}$ as a mixed number.

b. Write $1\frac{18}{21}$ as an improper fraction.

19. Find the sum of $157\frac{3}{10}$ and $103\frac{13}{15}$. Simplify the result.

20. Subtract and simplify, if possible: $67\frac{1}{4} - 29\frac{5}{6}$

21. Divide and simplify, if possible: $6\frac{1}{4} \div 3\frac{3}{4}$

22. BOXING Two of the greatest heavyweight boxers of all time are Muhammad Ali and George Foreman. Refer to the "Tale of the Tape" comparison shown below.

a. Which fighter weighed more? How much more?

b. Which fighter had the larger waist measurement? How much larger?

c. Which fighter had the larger forearm measurement? How much larger?

Tale of the Tape

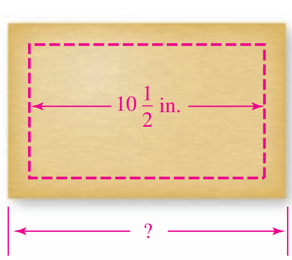
Muhammad Ali		George Foreman
6-3	Height	6-4
$210\frac{1}{2}$ lb	Weight	250 lb
82 in.	Reach	79 in.
43 in.	Chest (Normal)	48 in.
$45\frac{1}{2}$ in.	Chest (Expanded)	50 in.
34 in.	Waist	$39\frac{1}{2}$ in.
$12\frac{1}{2}$ in.	Fist	$13\frac{1}{2}$ in.
15 in.	Forearm	$14\frac{3}{4}$ in.

Source: The International Boxing Hall of Fame

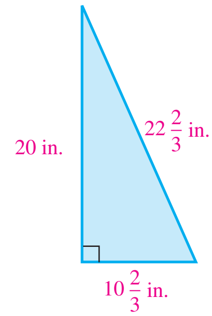
23. Evaluate the formula $P = 2l + 2w$ for $l = \frac{1}{3}$ and $w = \frac{1}{9}$.

24. SPORTS CONTRACTS A basketball player signed a nine-year contract for $\$13\frac{1}{2}$ million. How much is this per year?

25. SEWING When cutting material for a $10\frac{1}{2}$-inch-wide placemat, a seamstress allows $\frac{5}{8}$ inch at each end for a hem, as shown below. How wide should the material be cut to make a placemat?



26. Find the perimeter and the area of the triangle shown below.



27. NUTRITION A box of Tic Tacs contains 40 of the $1\frac{1}{2}$-calorie breath mints. How many calories are there in a box of Tic Tacs?

28. COOKING How many servings are there in an 8-pound roast, if the suggested serving size is $\frac{2}{3}$ pound?

29. Evaluate:

$$\left(\frac{2}{3} \cdot \frac{5}{16}\right) - \left(-1\frac{3}{5} \div 4\frac{4}{5}\right)$$

30. Evaluate: $\left(\frac{1}{2}\right)^3 \div \left(\frac{3}{4} - \frac{1}{3}\right)$

31. Simplify:

$$\frac{\frac{5}{6}}{\frac{7}{8}}$$

32. Simplify:

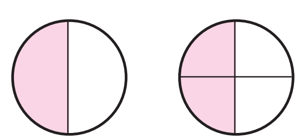
$$\frac{\frac{1}{2} + \frac{1}{3}}{-\frac{1}{6} - \frac{1}{3}}$$

33. Explain what is meant when we say, "The product of any number and its reciprocal is 1." Give an example.

34. Explain each mathematical concept that is shown below.

a. $\frac{6}{8} = \frac{\overset{1}{\cancel{2}} \cdot 3}{\underset{1}{\cancel{2}} \cdot 4} = \frac{3}{4}$

b.



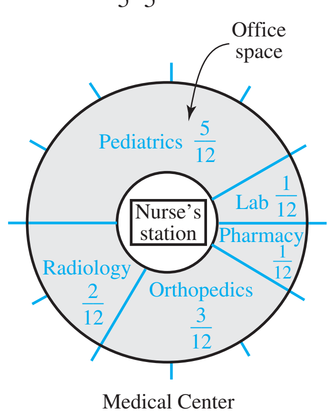
$\frac{1}{2} = \frac{2}{4}$

c. $\frac{3}{5} = \frac{3}{5} \cdot \frac{4}{4} = \frac{12}{20}$

Study Set Section 3.1 (page 216)

1. fraction **3.** proper, improper **5.** equivalent **7.** building **9.** equivalent fractions: $\frac{2}{6} = \frac{1}{3}$ **11. a.** improper fraction **b.** proper fraction **c.** proper fraction **d.** improper fraction **13.** 5 **15.** numerators **17.** $\frac{-7}{8}, -\frac{7}{8}$ **19.** 3, 1, 3, 18 **21.** numerator: 4; denominator: 5 **23.** numerator: 17; denominator: 10 **25.** $\frac{3}{4}, \frac{1}{4}$ **27.** $\frac{5}{8}, \frac{3}{8}$ **29.** $\frac{1}{4}, \frac{3}{4}$ **31.** $\frac{7}{12}, \frac{5}{12}$ **33. a.** 4 **b.** 1 **c.** 0 **d.** undefined **35. a.** undefined **b.** 0 **c.** 1 **d.** 75 **37.** $\frac{35}{40}$ **39.** $\frac{12}{27}$ **41.** $\frac{45}{54}$ **43.** $\frac{4}{14}$ **45.** $\frac{15}{30}$ **47.** $\frac{22}{32}$ **49.** $\frac{35}{28}$ **51.** $\frac{48}{45}$ **53.** $\frac{36}{9}$ **55.** $\frac{48}{8}$ **57.** $\frac{15}{5}$ **59.** $\frac{28}{2}$ **61. a.** no **b.** yes **63. a.** yes **b.** no **65.** $\frac{2}{3}$ **67.** $\frac{4}{5}$ **69.** $\frac{1}{3}$ **71.** $\frac{1}{24}$ **73.** $\frac{3}{8}$ **75.** in simplest form **77.** in simplest form **79.** $\frac{10}{11}$ **81.** $\frac{5}{9}$ **83.** $\frac{6}{7}$ **85.** $\frac{17}{13}$ **87.** $\frac{5}{2}$ **89.** $\frac{35}{12}$ **91.** $-\frac{1}{17}$ **93.** $-\frac{6}{7}$ **95.** $-\frac{8}{13}$ **97.** not equivalent **99.** equivalent **101. a.** 32 **b.** $\frac{5}{32}$ **103. a.** 16 **b.** $\frac{5}{8}$ **105. a.** 28, 22 **b.** $\frac{28}{50} = \frac{14}{25}$ **c.** $\frac{22}{50} = \frac{11}{25}$ **107. a.** 20 **b.** $\frac{2}{5}, \frac{3}{5}$

109.



117. $2,307

Study Set Section 3.2 (page 228)

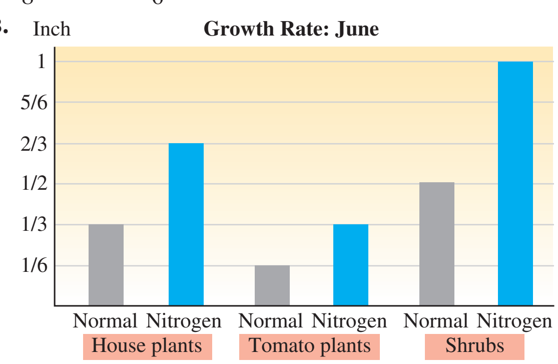
1. multiplication **3.** simplify **5.** area **7.** numerators, denominators, simplify **9. a.** negative **b.** positive **c.** positive **d.** negative **11.** base, height, $\frac{1}{2}bh$ **13. a.** $\frac{4}{1}$ **b.** $-\frac{3}{1}$ **15.** 7, 15, 2, 3, 5, 5, 24 **17.** $\frac{1}{8}$ **19.** $\frac{1}{45}$ **21.** $\frac{14}{27}$ **23.** $\frac{24}{77}$ **25.** $-\frac{4}{15}$ **27.** $-\frac{35}{72}$ **29.** $\frac{9}{8}$ **31.** $\frac{5}{2}$ **33.** $\frac{1}{2}$ **35.** $\frac{1}{7}$ **37.** $\frac{1}{10}$ **39.** $\frac{2}{15}$ **41. a.** $\frac{9}{25}$ **b.** $\frac{9}{25}$ **43. a.** $-\frac{1}{36}$ **b.** $-\frac{1}{216}$ **45.** $\frac{15}{32}$ **47.** 9 **49.** 15 ft^2 **51.** 63 $in.^2$ **53.** 6 m^2 **55.** 60 ft^2

57.

	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{12}$
$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{18}$
$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{16}$	$\frac{1}{20}$	$\frac{1}{24}$
$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{20}$	$\frac{1}{25}$	$\frac{1}{30}$
$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{24}$	$\frac{1}{30}$	$\frac{1}{36}$

59. $-\frac{1}{5}$ **61.** $\frac{21}{128}$ **63.** $\frac{1}{30}$ **65.** -15 **67.** $-\frac{27}{64}$ **69.** 1 **71.** $\frac{8}{3}$ **73.** $-\frac{3}{2}$ **75.** $\frac{2}{9}$ **77.** $-\frac{25}{81}$ **79.** $\frac{2}{3}$ **81.** $\frac{5}{6}$ **83.** $\frac{77}{60}$ **85.** $\frac{1}{2}$ **87.** 60 votes **89.** 18 in., 6 in., and 2 in. **91.** $\frac{3}{8}$ cup sugar, $\frac{1}{6}$ cup molasses

93.



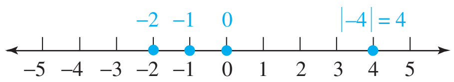
95. 27 ft^2 **97.** 42 ft^2 **99.** 9,646 mi^2 **101.** $\frac{3}{4}$ in. **109.** -2 **111.** 23

Study Set Section 3.3 (page 239)

1. reciprocal **3.** quotient **5. a.** multiply, reciprocal **b.** $\cdot$, $\frac{3}{2}$ **7. a.** negative **b.** positive **9. a.** 1 **b.** 1 **11.** 27, 27, 8, 9, 2, 4, 4, 9, 3 **13. a.** $\frac{7}{6}$ **b.** $-\frac{8}{15}$ **c.** $\frac{1}{10}$ **15. a.** $\frac{8}{11}$ **b.** -14 **c.** $-\frac{1}{63}$ **17.** $\frac{3}{16}$ **19.** $\frac{14}{23}$ **21.** $\frac{35}{8}$ **23.** $\frac{3}{4}$ **25.** 45 **27.** 320 **29.** -4 **31.** $-\frac{7}{2}$ **33.** $\frac{4}{55}$ **35.** $\frac{3}{23}$ **37.** 50 **39.** $\frac{5}{6}$ **41.** $\frac{2}{3}$ **43.** 1 **45.** $-\frac{5}{8}$ **47.** 36 **49.** $\frac{2}{15}$ **51.** $\frac{1}{192}$ **53.** $-\frac{27}{8}$ **55.** $-\frac{15}{2}$ **57.** $\frac{27}{16}$ **59.** $-\frac{1}{64}$ **61.** $\frac{3}{14}$ **63.** $\frac{8}{15}$ **65.** $\frac{13}{32}$ **67.** $\frac{2}{9}$ **69.** -6 **71.** $\frac{11}{6}$ **73.** $\frac{15}{28}$ **75.** $-\frac{5}{2}$ **77.** 4 applications **79.** 6 cups **81. a.** 30 days

b. 15 mi **c.** 25 days **d.** route 2 **83. a.** 16 **b.** $\frac{3}{4}$ in. **c.** $\frac{1}{120}$ in. **85.** 7,855 sections **93.** is less than **95.** Zero

97.



Think It Through (page 251)

$\frac{7}{20}$

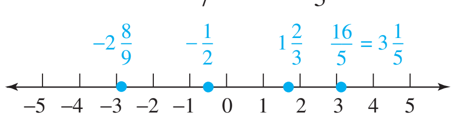
Study Set Section 3.4 (page 252)

1. common **3.** build, $\frac{2}{2}$ **5.** numerators, common, Simplify **7.** larger **9.** $\frac{9}{9}$ **11. a.** once **b.** twice **c.** three times **13.** 2, 2, 3, 3, 5, 180 **15.** 7, 7, 14, 35, 14, 5, 19 **17.** $\frac{5}{9}$ **19.** $\frac{1}{2}$ **21.** $\frac{4}{15}$ **23.** $\frac{2}{5}$ **25.** $-\frac{3}{5}$ **27.** $-\frac{5}{21}$ **29.** $\frac{3}{8}$ **31.** $\frac{7}{11}$ **33.** $\frac{10}{21}$ **35.** $\frac{9}{10}$ **37.** $\frac{1}{20}$ **39.** $\frac{13}{28}$ **41.** $\frac{1}{4}$ **43.** $\frac{1}{2}$ **45.** $-\frac{13}{9}$ **47.** $-\frac{3}{4}$ **49.** $\frac{19}{24}$ **51.** $\frac{31}{36}$ **53.** $\frac{24}{35}$ **55.** $\frac{9}{20}$ **57.** $\frac{3}{8}$ **59.** $\frac{4}{5}$ **61.** $\frac{11}{12}$ **63.** $\frac{7}{6}$ **65.** $\frac{2}{3}$ **67.** $\frac{11}{10}$ **69.** $\frac{1}{3}$ **71.** $\frac{22}{15}$ **73.** $\frac{2}{5}$ **75.** $-\frac{11}{20}$ **77.** $-\frac{3}{16}$ **79.** $\frac{1}{4}$ **81.** $\frac{23}{10}$ **83.** $\frac{5}{12}$ **85.** $\frac{341}{400}$ **87.** $\frac{9}{20}$ **89.** $\frac{20}{103}$ **91.** $-\frac{23}{4}$ **93.** $\frac{17}{54}$ **95.** $-\frac{1}{50}$ **97.** $\frac{5}{36}$ **99.** $-\frac{17}{60}$ **101. a.** $\frac{7}{32}$ in. **b.** $\frac{3}{32}$ in. **103.** $\frac{11}{16}$ in. **105. a.** $\frac{3}{8}$ **b.** $\frac{2}{6}=\frac{1}{3}$ **c.** $\frac{17}{24}$ of a pizza was left **d.** no **107.** $\frac{1}{16}$ lb, undercharge **109.** $\frac{7}{10}$ of the full-time students study 2 or more hours a day. **111.** no **113. a.** RR: right rear **b.** LR: left rear **117. a.** $\frac{3}{8}$ **b.** $\frac{1}{8}$ **c.** $\frac{1}{32}$ **d.** 2

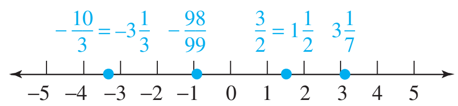
Study Set Section 3.5 (page 265)

1. mixed **3.** improper **5. a.** $5\frac{1}{3}°$ **b.** $-6\frac{7}{8}$ in. **7.** Multiply, Add, denominator **9.** $-\frac{4}{5}, -\frac{2}{5}, \frac{1}{5}$ **11.** improper **13.** not reasonable: $4\frac{1}{5}\cdot 2\frac{5}{7}\approx 4\cdot 3=12$ **15. a.** and, sixteenths **b.** negative, two **17.** 8, 4, 8, 4, 4, 4, 6, 6 **19.** $\frac{19}{8}, 2\frac{3}{8}$ **21.** $\frac{34}{25}, 1\frac{9}{25}$ **23.** $\frac{13}{2}$ **25.** $\frac{104}{5}$ **27.** $-\frac{68}{9}$ **29.** $-\frac{26}{3}$ **31.** $3\frac{1}{4}$ **33.** $5\frac{3}{5}$ **35.** $4\frac{2}{3}$ **37.** $10\frac{1}{2}$ **39.** 4 **41.** 2 **43.** $-8\frac{2}{7}$ **45.** $-3\frac{1}{3}$

47.



49.



51. $8\frac{1}{6}$ **53.** $7\frac{2}{5}$ **55.** 8 **57.** −10 **59.** $\frac{4}{9}$ **61.** $6\frac{9}{10}$ **63.** $2\frac{1}{3}$ **65.** $1\frac{10}{21}$ **67.** $-13\frac{3}{4}$ **69.** $-\frac{9}{10}$ **71.** $\frac{25}{9}=2\frac{7}{9}$ **73.** $2\frac{1}{2}$ **75.** 12 **77.** 14 **79.** −2 **81.** $-8\frac{1}{3}$ **83.** $\frac{35}{72}$ **85.** $\frac{5}{16}$ **87.** $-1\frac{1}{4}$ **89.** $-\frac{64}{27}=-2\frac{10}{27}$ **91. a.** $3\frac{2}{3}$ **b.** $\frac{11}{3}$ **93.** $2\frac{1}{2}$ **95. a.** $2\frac{2}{3}$ **b.** $-1\frac{1}{3}$ **97.** size 14, slim cut **99.** $76\frac{9}{16}$ in.^{2} **101.** $42\frac{5}{8}$ in.^{2} **103.** 64 calories **105.** 357¢ = $3.57 **107.** $1\frac{1}{4}$ cups **109.** 600 people **111.** $8\frac{1}{2}$ furlongs **115.** 60 **117.** 4

Think It Through (page 278)

workday: $6\frac{2}{3}$ hr; non-workday: $7\frac{5}{12}$ hr; $\frac{3}{4}$ hr

Study Set Section 3.6 (page 279)

1. mixed **3.** fractions, whole **5.** carry **7. a.** 76, $\frac{3}{4}$ **b.** $76+\frac{3}{4}$ **9. a.** 12 **b.** 30 **c.** 18 **d.** 24 **11.** 21, 5, 5, 35, 31, 35 **13.** $3\frac{7}{12}$ **15.** $6\frac{11}{15}$ **17.** $-2\frac{3}{8}$ **19.** $-3\frac{1}{6}$ **21.** $376\frac{17}{21}$ **23.** $714\frac{19}{20}$ **25.** $59\frac{28}{45}$ **27.** $132\frac{29}{33}$ **29.** $121\frac{9}{10}$ **31.** $147\frac{8}{9}$ **33.** $102\frac{13}{24}$ **35.** $129\frac{28}{45}$ **37.** $10\frac{1}{4}$ **39.** $13\frac{8}{15}$ **41.** $31\frac{14}{33}$ **43.** $71\frac{43}{56}$ **45.** $579\frac{4}{15}$ **47.** $62\frac{23}{32}$ **49.** $11\frac{1}{30}$ **51.** $5\frac{11}{30}$ **53.** $9\frac{3}{10}$ **55.** $3\frac{7}{8}$ **57.** $5\frac{2}{3}$ **59.** $10\frac{7}{16}$ **61.** $397\frac{5}{12}$ **63.** $-1\frac{11}{24}$ **65.** $7\frac{1}{2}$ **67.** $-5\frac{1}{4}$ **69.** $6\frac{1}{3}$ **71.** $53\frac{5}{12}$ **73.** $2\frac{1}{2}$ **75.** $-5\frac{7}{8}$ **77.** $3\frac{5}{8}$ **79.** $4\frac{1}{3}$ **81.** $461\frac{1}{8}$ **83.** $\frac{1}{4}$ **85.** $5\frac{1}{4}$ hr **87.** $7\frac{1}{6}$ cups **89.** $20\frac{1}{16}$ lb **91.** $108\frac{1}{2}$ in. **93.** $2\frac{3}{4}$ mi **95.** $48\frac{1}{2}$ ft **97. a.** 20¢ per gallon **b.** 20¢ per gallon **99.** $3\frac{1}{4}$ in. **105. a.** $4\frac{3}{4}$ **b.** $2\frac{1}{4}$ **c.** $4\frac{3}{8}$ **d.** $2\frac{4}{5}$

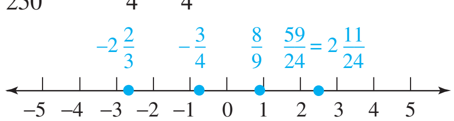
Study Set Section 3.7 (page 290)

1. operations **3.** complex **5.** raising to a power (exponent), multiplication, and addition **7.** $\left(\frac{2}{3}-\frac{1}{10}\right)+1\frac{2}{15}$ **9.** $\frac{2}{3}\div\frac{1}{5}$ **11.** $\frac{23}{4}$ **13.** 3, 6, 2, 2, 2, 5 **15.** $\frac{17}{20}$ **17.** $-\frac{1}{6}$ **19.** $-\frac{7}{26}$ **21.** $-\frac{1}{12}$ **23.** $5\frac{13}{30}$ **25.** $2\frac{2}{3}$ **27.** $26\frac{1}{4}$ **29.** 18 **31.** $\frac{5}{32}$ **33.** $\frac{5}{6}$ **35.** $\frac{5}{18}$ **37.** $-\frac{1}{2}$

39. $\frac{50}{13}$ **41.** $\frac{25}{26}$ **43.** $-1\frac{27}{40}$ **45.** $-1\frac{1}{3}$ **47.** 36 **49.** $\frac{1}{3}$ **51.** $\frac{31}{45}$ **53.** 5 **55.** $14\frac{5}{24}$ **57.** 11 **59.** $-1\frac{1}{6}$ **61.** $\frac{3}{7}$ **63.** $\frac{3}{10}$ **65.** $44\frac{1}{3}$ **67.** $8\frac{1}{2}$ **69.** $\frac{4}{9}$ **71.** $1\frac{37}{70}$ **73.** 3 **75.** $8\frac{4}{15}$ **77.** $91\frac{1}{4}$ in. **79.** yes **81.** $3\frac{1}{4}$ hr **83.** 9 parts **85.** 7 full tubes; $\frac{2}{3}$ of a tube is leftover **87.** 7 yd^2 **89.** 6 sec **95.** 2,248 **97.** 20,217 **99.** 1, 2, 3, 4, 6, 8, 12, 24

Chapter 3 Review (page 296)

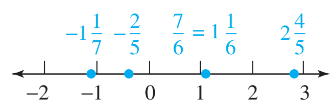
1. numerator: 11, denominator: 16; proper fraction **2.** $\frac{4}{7}, \frac{3}{7}$ **3.** The figure is not divided into equal parts. **4.** $-\frac{2}{3}, \frac{-2}{3}$ **5. a.** 1 **b.** 0 **c.** 18 **d.** undefined **6.** equivalent fractions: $\frac{6}{8} = \frac{3}{4}$ **7.** $\frac{12}{18}$ **8.** $\frac{6}{16}$ **9.** $\frac{21}{45}$ **10.** $\frac{65}{60}$ **11.** $\frac{45}{9}$ **12. a.** no **b.** yes **13.** $\frac{1}{3}$ **14.** $\frac{5}{12}$ **15.** $\frac{11}{18}$ **16.** $\frac{9}{16}$ **17.** in simplest form **18.** equivalent **19.** $\frac{7}{24}, \frac{17}{24}$ **20. a.** The fraction $\frac{5}{8}$ is being expressed as an equivalent fraction with a denominator of 16. To build the fraction, multiply $\frac{5}{8}$ by 1 in the form of $\frac{2}{2}$. **b.** The fraction $\frac{4}{6}$ is being simplified. To simplify the fraction, remove the common factors of 2 from the numerator and denominator. This removes a factor equal to 1: $\frac{2}{2} = 1$. **21.** numerators, denominators, simplify **22.** $\frac{5}{6} \cdot \frac{2}{3}$ **23.** $\frac{1}{6}$ **24.** $-\frac{14}{45}$ **25.** $\frac{5}{12}$ **26.** $-\frac{1}{25}$ **27.** $\frac{21}{5}$ **28.** $\frac{9}{4}$ **29.** 1 **30.** 1 **31.** $-\frac{9}{16}$ **32.** $-\frac{125}{8}$ **33.** $-\frac{8}{125}$ **34.** $\frac{4}{9}$ **35.** 2 mi **36.** 30 lb **37.** 60 in.^{2} **38.** 165 ft^2 **39. a.** 8 **b.** $-\frac{12}{11}$ **c.** $\frac{1}{5}$ **d.** $\frac{7}{8}$ **40.** multiply, reciprocal **41.** $\frac{25}{66}$ **42.** $-\frac{7}{8}$ **43.** $\frac{6}{5}$ **44.** $\frac{30}{7}$ **45.** $-\frac{3}{2}$ **46.** $\frac{8}{5}$ **47.** $-\frac{1}{180}$ **48.** 1 **49.** 12 pins **50.** 30 pillow cases **51.** $\frac{5}{7}$ **52.** $\frac{1}{2}$ **53.** $\frac{5}{4}$ **54.** $-\frac{6}{5}$ **55. a.** $\frac{5}{8}$ **b.** $\frac{1}{5}$ **56.** 2, 3, 3, 5, 90 **57.** $\frac{5}{6}$ **58.** $-\frac{31}{40}$ **59.** $\frac{19}{48}$ **60.** $\frac{20}{7}$ **61.** $-\frac{23}{36}$ **62.** $\frac{7}{12}$ **63.** $-\frac{23}{6}$ **64.** $\frac{47}{60}$ **65.** $\frac{7}{32}$ in. **66.** $\frac{3}{4}$ **67.** the second hour: $\frac{3}{11} > \frac{2}{9}$ **68.** $\frac{1}{250}$ **69.** $4\frac{1}{4} = \frac{17}{4}$ **70.**



71. $3\frac{1}{5}$ **72.** $-3\frac{11}{12}$ **73.** 17 **74.** $2\frac{1}{3}$ **75.** $\frac{75}{8}$ **76.** $-\frac{11}{5}$ **77.** $\frac{53}{14}$ **78.** $\frac{199}{100}$ **79.** $2\frac{1}{10}$ **80.** $-\frac{21}{22}$ **81.** 40 **82.** $2\frac{1}{2}$ **83.** 16 **84.** $-40\frac{4}{5}$ **85.** $7\frac{9}{16}$ **86.** $6\frac{2}{9}$ **87.** $48\frac{1}{8}$ in. **88.** 87 in.^{2} **89.** 40 posters **90.** 9 loads **91.** $3\frac{23}{40}$ **92.** $6\frac{1}{6}$ **93.** $1\frac{1}{12}$ **94.** $1\frac{5}{16}$ **95.** $255\frac{19}{20}$ **96.** $23\frac{32}{35}$ **97.** $83\frac{1}{18}$ **98.** $113\frac{7}{20}$ **99.** $31\frac{11}{24}$ **100.** $316\frac{3}{4}$ **101.** $20\frac{1}{2}$ **102.** $34\frac{3}{8}$ **103.** $39\frac{11}{12}$ gal **104.** $\frac{5}{8}$ in. **105.** $\frac{8}{9}$ **106.** $\frac{19}{72}$ **107.** $8\frac{8}{15}$ **108.** $-3\frac{5}{8}$ **109.** $-\frac{12}{17}$ **110.** $\frac{26}{29}$ **111.** $-\frac{2}{5}$ **112.** $\frac{63}{17}$ **113.** $2\frac{23}{40}$ **114.** $14\frac{1}{16}$ **115.** $8\frac{1}{3}$ **116.** $11\frac{1}{6}$ **117.** 5 full tubes, $\frac{9}{10}$ of a tube is left over **118.** 8 in.

Chapter 3 Test (page 311)

1. a. numerator, denominator **b.** equivalent **c.** simplest **d.** simplify **e.** reciprocal **f.** mixed **g.** complex **2. a.** $\frac{4}{5}$ **b.** $\frac{1}{5}$ **3.** $\frac{13}{6} = 2\frac{1}{6}$ **4.**



5. yes **6.** $\frac{21}{24}$ **7. a.** 0 **b.** undefined **8. a.** $\frac{3}{4}$ **b.** $\frac{2}{5}$ **9.** $\frac{5}{8}$ **10.** $-\frac{3}{20}$ **11.** 6 **12.** $\frac{11}{20}$ **13.** $\frac{11}{7}$ **14.** $\frac{1}{3}$ **15.** $\frac{9}{10}$ **16.** 40 **17.** $\frac{47}{50}$ **18. a.** $9\frac{1}{6}$ **b.** $\frac{39}{21}$ **19.** $261\frac{1}{6}$ **20.** $37\frac{5}{12}$ **21.** $1\frac{2}{3}$ **22. a.** Foreman, $39\frac{1}{2}$ lb **b.** Foreman, $5\frac{1}{2}$ in. **c.** Ali, $\frac{1}{4}$ in. **23.** $\frac{8}{9}$ **24.** $\$1\frac{1}{2}$ million **25.** $11\frac{3}{4}$ in. **26.** perimeter: $53\frac{1}{3}$ in., area: $106\frac{2}{3}$ in.^{2} **27.** 60 calories **28.** 12 servings **29.** $\frac{13}{24}$ **30.** $\frac{3}{10}$ **31.** $\frac{20}{21}$ **32.** $-\frac{5}{3}$ **33.** When we multiply a number, such as $\frac{3}{4}$, and its reciprocal, $\frac{4}{3}$, the result is 1: $\frac{3}{4} \cdot \frac{4}{3} = 1$ **34. a.** removing a common factor from the numerator and denominator (simplifying a fraction) **b.** equivalent fractions **c.** multiplying a fraction by a form of 1 (building an equivalent fraction)

Chapters 1–3 Cumulative Review (page 313)

1. a. 5 **b.** 8 hundred thousands **c.** 5,896,600 **d.** 5,900,000 **2.** hundred billions **3.** Orange, San Diego, Kings, Miami-Dade, Dallas, Queens **4. a.** 450 ft **b.** 11,250 ft^2 **5.** 30,996 **6.** 16,544, 16,544 + 3,456 = 20,000 **7.** 2,400 stickers **8.** 299,320 **9.** 991, $991 \cdot 35 = 34{,}685$